Is the KOSPI200 Options Market Efficient? Parametric and Nonparametric Tests of the Martingale Restriction

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Abstract

A number of studies on the S&P 500 index options market claim that the no arbitrage assumption cannot be rejected for this market because either the martingale restriction defined in Longstaff (1995) cannot be rejected by the data, or, even when it is rejected, a large proportion of the violation can be explained by market friction factors. The present study singles out the effect of market inefficiency from market friction by testing the martingale restriction for the KOSPI 200 index options market, which is the most liquid and active options market in the world. Not only using the parametric methods adopted in previous studies but also using the nonparametric methods which enable us to avoid the model misspecification problem, we empirically present clear evidence of a violation of the martingale restriction. In addition, in contrast to the S&P 500 options market, regression analyses and robustness tests indicate that market friction factors can explain only a small portion of the percentage differences between option-implied and market-observed index prices. Overall, the results do not support the basic no-arbitrage assumption or the market efficiency in the KOSPI 200 options market.

1. INTRODUCTION

One of the most important arguments under no-arbitrage option pricing theory is that if a financial market is frictionless and allows for no arbitrage opportunities, then there exists at least one risk-neutral probability measure under which the underlying asset price discounted at the risk-free rate follows the martingale process. Therefore, a violation of this property—first termed "internal consistency" by Harrison and Kreps (1979) and later referred to as "martingale restriction" by Longstaff (1995)—in a financial market can be attributed to the existence of arbitrage opportunities and/or market friction. Both have important implications for the popular risk-neutral valuation approach in derivatives pricing and regulators of the corresponding market, and thus, testing the martingale restriction is a matter of great importance for scholars, practitioners, and policy makers.

One obvious way to test the validity of the martingale restriction is to compare the market-observed price of the underlying asset with the price implied from its derivatives (e.g., options contracts). Option-implied price is normally computed as the (discounted) first moment of the risk neutral density (RND) of the underlying asset. Depending on the assumptions made about the functional form of the RND, previous studies typically adopt one of two approaches. The first extracts the spot price based on specific option pricing models, mainly the Black-Scholes (BS) model, which assumes a lognormal RND. However, a rejection of the martingale restriction based on the simple BS model remains questionable because the log-normality assumption is too restrictive. Therefore, the second approach relaxes this assumption to allow for more general forms of the density function. For example, Longstaff (1995) adopts a four-parameter Hermite polynomial to approximate the RND and estimates parameters by using four call prices closest to the money. Alternatively, Brenner and Eom (1997) consider a Laguerre polynomial series as an approximation of the density function. They show that, the Hermite polynomial expansion in Longstaff (1995) can lead to pricing bias if the true density function is not lognormal.

No matter which of the two above approaches is taken, the RND used for testing the martingale restriction is parametrically defined, and hence is subject to the model misspecification problem. To obtain a rough estimate of the impact of model misspecification on the results, Brenner and Eom (1997) report that the Laguerre density estimator can reduce the mean percentage difference by approximately 80% more than the lognormal specification and approximately 10% more than the Hermite polynomial. Strong and Xu (1999) also report that the percentage price difference is about 8 times larger under the lognormal than under a generalized RND. To avoid this problem, the present study applies the recent advances in the nonparametric estimation of the RND from option prices (see Bahra, 1997; Jackwerth, 1999; Figlewski, 2008 for detailed reviews). The nonparametric methods adopted in this study are the kernel regression method described in Aït-Sahalia and Lo (1998) and the implied volatility smoothing method in Shimko (1993) because of their advantages over other nonparametric methods. For comparison, we also investigate the martingale restriction by using parametric methods based on the BS model, the Hermite polynomial RND in Longstaff (1995) and the Laguerre polynomial RND in Brenner and Eom (1997). To the best of our knowledge, this study is the first to examine the martingale restriction property using nonparametric methods.

Empirical results of testing the martingale restriction so far have been mixed and the validity of the restriction seems to vary across markets. Manaster and Rendleman (1982) and Longstaff (1995) examine the S&P 100 options market and claim strong evidence of a violation of the martingale restriction. To avoid the American and wild-card features of the S&P 100 options market, Strong and Xu (1999) conduct a similar test using S&P 500 index calls and puts over the 1990-1994 period and claim an economically insignificant rejection of the martingale restriction. Brenner and Eom (1997) also examine the S&P 500 options market and they fail to reject the martingale restriction. On the other hand, Neumann and Schlag (1996) consider the DAX options market and reject the martingale restriction for the first half of 1994. Turvey and Komar (2006) test a variation of the martingale restriction for the live cattle options traded on the Chicago Mercantile Exchange (CME) and find that the option-implied market price of risk varies systematically across strike prices and randomly over time, suggesting a clear violation of the martingale restriction.

In cases of the rejections, interpreting the results is important. Longstaff (1995) implements a regression analysis, which controls for the moneyness, time-to-maturity, and volatility biases of the

BS model. He demonstrates that the percentage differences are largely due to market friction factors, including bid-ask spreads of option prices, trading volume, and other liquidity factors. Brenner and Eom (1997) and Neumann and Schlag (1996) make similar conclusions that violations of the martingale restriction in their respective markets are also due to market frictions. Based on these observations, a general conclusion in the current literature is that no arbitrage coupled with a general form of distribution is a reasonable assumption for option pricing in these markets.

Unlike the above studies, this study considers the KOSPI 200 options market, which is the single most liquid and actively traded derivatives market in the world. The motivation is obvious: given the negligible frictions in this market, if the martingale restriction ever gets rejected, then it is most likely due to the failure of the no-arbitrage assumption. Hence, testing the martingale restriction for this market offers a unique opportunity to test for market efficiency. This study is also motivated by the fact that previous studies generally employ outdated data (up to 1994), and global financial markets and the nature of their microstructures have undergone substantial changes over the last two decades. Thus, the martingale restriction needs to be tested by using more recent data.

The empirical results of this study clearly reject the martingale restriction for the 2002-2010 period. Furthermore, a regression of percentage differences between option-implied and market-observed underlying prices on market friction factors indicates that, inconsistent with developed markets such as the S&P 500 and DAX options markets, the differences in the KOSPI 200 options market are related more to speculation than to market friction, casting doubt on the efficiency of this market. The results and conclusions are robust to the use of KOSPI 200 index futures instead of the KOSPI 200 index as the underlying asset.

The rest of the paper is organized as follows. Section 2 briefly reviews the theory underlying the empirical test and summarizes the parametric and nonparametric methods used in this study. Section 3 describes the property of the KOSPI 200 options market and discusses the data. Section 4 presents the empirical results for the martingale restriction. A robustness check is undertaken in Section 5 and Section 6 concludes this study.

2. THEORY AND METHODOLOGY

2.1 Review of Theory

Consider a financial market over a finite investment horizon (0, T). The market has one zero coupon bond with a riskless continuous compounding interest rate r[1], one unit of traded underlying asset S_t and derivatives with payoff functions $F(S_T)$. The probabilistic structure of the market is based on a probability space (Ω, B, P), where Ω is the payoff space of the underlying asset, B the sigma algebra generated by the payoffs in Ω, and P an objective probability measure assigned on

[1] This study calculates the risk-free rate by using the 91-day certificate deposit (CD) rate.

(Ω, B, P). Let I_t denote the information set faced by investors at time t, then the payoff space $\Gamma_t = \{\pi_t \in I_t : E[\pi_t^2 | I_{t-1}] < \infty\}$ where π_t denotes an asset payoff at time t. That is, the payoff space Γ_t is the set of all random variables with finite conditional second moments given the previous period information. Attention is restricted to derivatives with payoff $F(S_t) \in \Gamma_t$.

Harrison and Kreps (1979) show that to avoid arbitrage the pricing operator mapping the date-T payoffs to date-0 prices can be characterized as an expectation operator by Riesz representation theorem. That is, under the frictionless-market and no-arbitrage assumptions, the price H of a derivative asset is given as follows:

$$H = E_P[\zeta(t,T) F(S_T) | I_t] = \int_0^\infty \zeta(t,T) F(S_T) f_P(S_T | I_t) dS_T \qquad (1)$$

where $\tau = T - t$ is the time to maturity, S_T the price of the underlying asset at the maturity date T, $\xi(t,T)$ a market pricing kernel function, and $f_P(S_T | I_t)$ the date-t probability density for the date-T payoff under the probability measure P. Since $e^{r\tau}\xi(t,T)$ is non-negative, square-integrable, and has an expected value of one, a direct application of the Radon-Nikodym theorem changes the original measure P to a new measure Q and simplifies equation (1) to the following:

$$H = E_Q[e^{-r\tau} F(S_T) | I_t] = e^{-r\tau} \int_0^\infty F(S_T) f_Q(S_T | I_t) dS_T \qquad (2)$$

The new probability measure Q is called a risk neutral measure if and only if the underlying asset can be priced by the pricing equation (2), i.e. the underlying price is internally consistent:

$$S_t = E_Q[e^{-r\tau} S_T | I_t] = e^{-r\tau} \int_0^\infty S_T f_Q(S_T | I_t) dS_T \qquad (3)$$

That is, under the market frictionless assumption, the derivatives-implied underlying asset price should equal the actual market value of the underlying asset. This is the definition of the martingale restriction proposed in Longstaff (1995). Obviously, a violation of the martingale restriction then implies the existence of arbitrage opportunities and/or market friction. Our approach to testing the martingale restriction involves calibrating (parametrically) or estimating (non-parametrically) the risk-neutral density function of the underlying from option prices by using Equation (2) and then computing Equation (3) to compare the option-implied price of the underlying with the market price.

We classify the methods for estimating the option-implied risk-neutral density into two categories—parametric and nonparametric methods—based on whether a specific form of the risk-neutral density is assumed. In later sections, we use the words “methods” and “models” interchangeably.

2.2 Parametric Methods

We select the classical BS model, Longstaff's (1995) four-factor Hermite polynomial and Brenner and Eom's (1997) Laguerre polynomial series to represent the parametric methods. European options under the BS model are valued as follows:

$$C=N(d_1)S_t-N(d_2)Ke^{-r\tau},\ P=N(-d_2)Ke^{-r\tau}-N(-d_1)S_t \tag{4}$$

where C and P denote the prices of call and put options, respectively, $N(.)$ is the cumulative distribution of a standard normal distribution, σ the volatility of the return on the underlying asset, S_t the price of the dividend-adjusted[2] underlying asset at time t, $d_1=[ln(S_t/K)+(r+\sigma^2/2)\tau]/\sigma\sqrt{\tau}$, and $d_2=d_1-\sigma\sqrt{\tau}$. Then the spot price implied by the BS model can be calibrated and compared with the market price of underlying assets. Realizing that the BS model may be subject to the problem of model misspecification, Longstaff (1995) extends the lognormal density assumption to a four-factor Edgeworth expansion, a general form that includes the BS model, Merton's (1973) stochastic interest rate model, and Merton's (1976) jump diffusion model as its special cases.

Let $Z=(ln(S_T)-\mu)/\sigma$, where μ is the conditional mean. The risk-neutral density of Z is assumed to belong to an Edgeworth expansion family of density functions:

$$q(Z)=\{1+\beta(Z^3-3Z)+\gamma(Z^4-6Z^2+3)\} \tag{5}$$

where β and γ are parameters indicating the skewness and kurtosis of the price of the underlying asset. Note that Equation (5) reduces to the standard normal density in the BS model when both β and γ are set to zero. A call option is priced as follows:

$$\mathrm{C}=e^{-r\tau}\int_0^{\infty}max(0,\ e^{\mu+\sigma Z}-K)q(Z)dZ \tag{6}$$

The price of a put option can be derived based on the put-call parity

$$P=Ke^{-r\tau}-S_t+C \tag{7}$$

Once the parameters are estimated, the first moment of the risk-neutral density is given by

$$E_t[S_T]=exp(\mu+\sigma^2/2)(1+\beta\sigma^3+\gamma\sigma^4) \tag{8}$$

and the implied underlying price is expressed as follows:

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[2] In this study, S_t always denotes the dividend-adjusted spot price.

$$S_t=exp(-r\tau+\mu+\sigma^2/2)(1+\beta\sigma^3+\gamma\sigma^4) \quad (9)$$

Brenner and Eom (1997) approximate the density function of the underlying asset by a Laguerre polynomial series instead of Longstaff's (1995) Hermite polynomial series. They derive the equation for a call option as

(10)

In Equation (10), , $\mu_j=E_t(S_T^j)$, $\theta=VAR(S_T)/\mu_1$, $\delta=\mu_1/\theta$, $I(.)$ is an incomplete gamma function, and $\Gamma(.)$ is a gamma function. The put option value can be calculated using the put-call parity. By using up to the fourth order (k=4), Brenner and Eom (1997) estimate the parameters and the implied underlying price as $S_t=exp(-r\tau)\mu_1$.

2.3 Nonparametric Methods

Breeden and Litzenberger (1978) show that the risk-neutral density $f_t(S_T)$ can be obtained from option prices by taking the second-order derivative of the option pricing function with respect to the strike price K:

$$\hat{f}_t(S_T) = e^{r\tau}\left[\frac{\partial^2 H}{\partial K^2}\right]_{(\square|S_T=K)} \quad (11)$$

A number of nonparametric methods have been proposed for estimating the option pricing function H. In this study, we choose kernel regression and implied volatility smoothing as the main nonparametric methods. As shown in Ait-Sahalia and Lo (1998), kernel regression is better suited for hypothesis testing and requires less restrictive assumptions than other methods do. Shimko (1993) argues that fitting/smoothing the implied volatility curve is more accurate and convenient than fitting/smoothing the pricing function directly because the implied volatility curve is better behaved than the pricing function.

Given a set of historical European option prices H_i and accompanying variables V_i=(S_i, K_i, r_i, τ_i),[3] let h be the bandwidth used for the kernel regression. The estimator of H conditional on V is given as the Nadaraya-Watson kernel estimator:

(12)

where n is the sample size and $k((V-V_i)/h)$ the kernel function. In this study, we choose the second-order Gaussian kernel[4] as

(13)

and choose h by minimizing the least squares cross-validation.[5]

Because we have four explanatory variables, our multivariate kernel function becomes the product of four univariate kernels[6], giving the following full nonparametric pricing function[7]

(14)

Differentiating the function twice numerically generates the risk-neutral density implicit in option prices.

On the other hand, Shimko (1993) smoothes the implied volatility structure by the parabolic function of the strike price:

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[3] For notational convenience, we suppress the time variable t.

[4] The choice of kernel function or kernel order usually affects the speed of convergence of the estimator to the true function, but typically has little influence on the end result, whereas that of the bandwidth h determines the accuracy of the final outcome. See Ait-Sahalia and Lo (1998) for a discussion. Therefore we use the popular Gaussian kernel for the whole analysis and optimize the parameter h for each data set.

[5] See, for example, Stone (1984).

[6] The kernel function $k(V_i)$ can take a multidimensional form and use the same bandwidth h for all variables V_i=(S_i, K_i, r_i, τ_i). Alternatively, a multiplicative kernel $k(V_i)=k_S(S_i)\times k_K(K_i)\times k_r(r_i)\times k_\tau(\tau_i)$ can be assumed where each univariate kernel has its own bandwidth h. Uller (1997) analyzes the asymptotics of this latter solution. Ait-Sahalia and Lo (1998), and Li and Zhao (2009) use it for S&P 500 options and interest rate caps. The option pricing error analysis in Section 5.3 demonstrates its accuracy.

[7] We also test the kernel regression method by fitting implied volatilities instead of option prices because this allows us to reduce the dimension of the nonparametric pricing function from four to three. We obtain similar results.

$$\sigma(K,\tau)=A_0(\tau)+A_1(\tau)K+A_2(\tau)K^2 \tag{15}$$

By fitting the parameters A_0, A_1, and A_2 with least squares, we can generate a smooth volatility curve for all K and τ. The smoothed volatility is then used to compute a series of option prices via the BS equation,[8] and the risk-neutral density function is solved analytically as follows:

$$f_t(S_T)=-n(d_2)[d_{2K}-(A_1+2A_2K)(1-d_2d_{2K})-2A_2K] \tag{16}$$

where $d_{1k}=(-1/Kv)+(1-d_1/v)(A_1+2A_2K)$, $d_{2K}=d_{1K}-(A_1+2A_2K)$, $v=\sigma\sqrt{\tau}$, $d_1$and d_2 are the same as those in the BS equation and $n(.)$ is the density distribution of a standard normal variable.

3. THE KOSPI 200 OPTIONS MARKET AND SAMPLE DATA

As discussed in the Introduction section, testing the martingale restriction for the KOSPI 200 index options market provides a unique opportunity for testing market efficiency because of its vast liquidity and negligible transaction costs. Despite its short history, the KOSPI 200 options market has become the single most active derivatives market in the world. As shown in Table 1, which presents the trading volumes (in terms of the number of contracts) for the top 10 equity index futures and options products and their respective exchanges for the recent year of the sample data (i.e., 2010), the KOSPI 200 options market clearly dominates others in terms of trading activity.

In addition to its ample liquidity, the KOSPI 200 options market requires negligible transaction costs. For equity trading, individuals should pay substantial taxes and commissions for trading, and institutions should pay membership and exchange fees. By contrast, for options trading, individual investors are exempt from taxes and often do not pay commissions.[9] In addition, membership and exchange fees are lower in the options market. Further, in the options market, the bid-ask spread, which is widely used to measure transaction costs and market liquidity, is very narrow—typically one or two times the minimum tick size (see Ahn, Kang & Ryu, 2008, 2010).

Interestingly, the KOSPI 200 options market is also known for highly speculative options traders who tend to be easily influenced by behavioral and psychological biases, which may lead to systematic pricing bias in the implied spot price even with no market friction. Sophisticated and informed institutional investors are dominant market players in developed countries' derivatives

[8] Note that Shimko (1993) does not require the BS model to be correct. The equation is used only to interpolate implied volatility, not to determine the market price of options.

[9] Investment firms that provide individual investors with a home trading system typically require very small commissions. Recently, an increasing number of firms have been waiving such commissions.

markets, whereas domestic individuals are very active investors in the KOSPI 200 options market. As shown in Table 2, which presents trading volume by investor type, domestic individual investors account for more than 42% of the total trading volume for the sample period. Further, these individual investors focus on out-of-the-money (OTM) and especially deep-OTM options, which are typically used as high-leverage speculative trading tools (see Ahn et al., 2008, 2010).[10] Again, this coexistence of extreme liquidity, low transaction costs, and abundant speculative activity in the KOSPI 200 options market raises the question of whether these imply the existence of arbitrage opportunities in the case of a violation of the martingale restriction for the market.

The maturity dates of KOSPI 200 options are set as the second Thursday of three consecutive near-term months and one nearest month from the quarterly cycle (March, June, September, and December). For instance, if today is April 1, then the maturity months are April, May, June, and September. Although these four types of option contracts (classified by their maturity dates) are always listed, only the nearest maturity contracts are actively traded on the Korea Exchange (KRX) (see Ryu, 2011).[11] Therefore, daily data on only the nearest maturity contracts from January 2002 to December 2010 are considered in this study. Then options data are matched with these daily index prices. For each trading day, option contracts are screened with the following six filters. First, options with fewer than 30 intraday transactions are excluded. Second, bid and ask prices should exceed the minimum tick size. Third, closing prices of options should exceed the no-arbitrage lower bounds and lie between 0.03 and 25 points. Equation (17) (Equation (18)) represents the no-arbitrage lower bound for call (put) options:

$$C_t \geq Max(S_t\text{-}Ke^{-r\tau},0) \quad (17)$$

$$P_t \geq Max(Ke^{-r\tau}\text{-}S_t,0) \quad (18)$$

Fourth, BS implied volatility should be greater than 0.05 and less than 0.95. Fifth, the absolute value of option moneyness, classically defined as $|K/S_t\text{-}1|$, should be less than 0.1. Finally, we exclude those trading days with fewer than 8 option contracts.

Table 3 summarizes the daily statistics for the KOSPI 200 index and the call and put options. The average (median) number of calls and puts traded daily is 9.97 (9) and 10.68 (10),

[10] The Korean options market adopts two different tick sizes based on option prices. If the option price exceeds 3 points (i.e., KRW 300,000), then the minimum tick size is 0.05 points (i.e., KRW 5,000), whereas the tick size is 0.01 points (i.e., KRW 1,000) if the option price is less than 3 points.

[11] In an analysis not discussed in this study, we further exclude those contracts with less than three days to the next maturity date and re-test all models. The main conclusions remain, indicating that even when obviously speculative market behavior, which is often found around maturity dates, is excluded, ample arbitrage activity remains in the market.

respectively. The average moneyness[12] is negative for calls (-0.618), but positive for puts (1.344). Because only the nearest maturity contracts are used, both calls and puts have a relatively short time to maturity. The maximum time to maturity is 0.0984 years (approximately a month), and the median time to maturity is 0.0407 years (approximately two weeks). Over the sample period, the KOSPI 200 index fluctuates from the minimum value of 65.58 to the maximum value of 168.23.

4. EMPIRICAL RESULTS AND DISCUSSION

4.1 Tests of the Martingale Restriction

We test the martingale restriction by using the methods described in Section 2. As shown in Table 4, the results clearly indicate a violation of the martingale restriction for the sample. The null hypothesis for call options is rejected by all methods at the 99% level of significance, and for put options at the 95% level of significance. Three out of five models report positive mean and median percentage differences for call options, and two models report positive mean differences for put options.

More specifically, the BS model has an average percentage difference of 0.141% for calls and 0.206% for puts. The Longstaff model gives the smallest mean absolute percentage differences (-0.058% for calls and -0.032% for puts) among the parametric methods. Between the nonparametric methods, the Shimko method provides smaller mean percentage differences: 0.032% for calls and 0.029% for puts, indicating a price difference of only 0.06 points. However, this is still substantial in that spreads in the options market are often just a tick (i.e., 0.01 points). The kernel regression (KR) method produces the largest mean percentage difference for calls (1.291%) and the second largest difference for puts (-1.183%). As shown in Table 5, which lists those parameters for both kernel regressions on calls and puts, the high R^2 values demonstrate the accuracy of the kernel regression method for option price estimation.

The results in Table 4 suggest that it is more expensive to trade the underlying asset on the options market than to trade it directly on the spot market. Longstaff (1995) explains that these relatively "expensive" option prices may reflect higher transaction costs in options markets. However, this explanation may not be applied to the KOSPI 200 options market because its bid-ask spread is very narrow and transaction costs (e.g., commissions, taxes, and transaction fees) are negligible. We further investigate this conjecture in Section 4.2.

4.2 The Role of Market Friction in Price Differences

Option-implied prices of underlying assets can be different from market-observed spot prices because of market friction. Longstaff (1995) regresses differences between implied and observed

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[12] Here, the moneyness for call (put) options is defined as the average difference between their index value (strike) and strike price (index value) for a given day.

index prices on several variables for market friction, including the average bid-ask spread, open interest, and total option trading volume, while controlling for the time to maturity, moneyness, and the current and first two lagged values of absolute daily returns on the S&P 100 index. He finds that all market friction variables are significant and that the R^2 value is as high as 64.4%, demonstrating that market friction factors are major reasons behind the violation of the martingale restriction. By contrast, using S&P 500 options data, Brenner and Eom (1997) find no relationship between estimated price differences and market friction for their proposed general distribution of the risk-neutral density.

In this section, we regress the percentage differences between implied and observed underlying prices on several measures of market friction for KOSPI 200 options, including the average bid-ask adjusted spread, total open interest, total trading volume, and the total number of options used to compute price differences.[13] Table 6 reports the results. The estimate for bid-ask adjusted spread is insignificant for the BS and Longstaff model for call options and for the Shimko models for put options. However, the coefficient is negative for call options and positive for put options. The coefficient for total open interest is negative and significant for four out of five models for call options, and three models for put options at the 99% significance level. The negative sign is as expected because as the total open interest increases, the options market becomes more liquid. The total trading volume has a similar significance level with the total open interest, but has coefficients with mixed sign. The sign of the coefficient for the number of options is mixed with most being positive, which is inconsistent with the significant negative relationship reported in Longstaff (1995). The kernel regression model shows the highest R^2 values: 24.6% for calls and 36.4% for puts, followed by the BS model (20.2% for calls and 12.5% for puts). The Brenner and Eom model has the lowest R^2 value: 1.2% for calls, and 1.4% for puts, indicating the poor explanatory power of these market friction variables.

To further verify the validity of our results, following Longstaff (1995), we include several control variables in the regression to capture any potential model biases: the average value of the time to maturity for the time-to-expiration bias; the average value of moneyness for the option leverage bias; and the current, first, and second lagged values of absolute daily returns on the KOSPI 200 index for the volatility bias. Table 7 reports the regression coefficients and *p*-values for call and put options. As expected, including these variables does not significantly increase the R^2 value. The Longstaff, Brenner and Eom, and Shimko models all show an adjusted R^2 value less than 10%. Further, the results indicate that when these biases are controlled for, the explanatory performance of the market friction variables is weakened in general. For instance, the signs of the coefficients for both bid-ask adjusted spread and total open interest become mixed.

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[13] The bid-ask spread is adjusted by dividing it by the option price. Scaling the spread is more meaningful given the narrow spread for KOSPI 200 options.

In sum, our regression analysis shows that, at best, the market friction only marginally explains the violation of the martingale restriction for the KOSPI 200 options market. This raises several possible interpretations. First, one may argue that any test of the martingale restriction should consider how often the implied index price falls within the bid-ask spread of the underlying asset. However, considering that the KOPSI200 index consists of the most liquid 200 stocks in Korea and that their bid-ask spreads are usually quite narrow, this may not be a valid argument. In addition, a trader attempting to take advantage of a violation of the martingale restriction is likely to long the underlying asset and simultaneously short a synthesized underlying asset by using options and risk-free assets, while a short-seller may take the opposite position. Hence, the bid-ask spread of the underlying asset is already reflected in the option price, rendering this argument irrelevant. Second, one may claim that the KOSPI 200 options market's ample liquidity may induce options traders to willingly accept higher premiums. However, as already shown in Table 6, with the effects of total open interest and the total trading volume of KOSPI 200 options controlled for, the violation of the martingale restriction is still little explained. Third, crash-averse institutional investors' excess demand may drive up option prices. Institutional investors may buy OTM puts to hedge against potential crashes in futures markets. However, this argument does not explain the low adjusted R^2 value in Table 7, in which we control for the effects of moneyness and the number of puts traded on each particular day.

The above discussion leads back to the initial conjecture proposed in the Introduction section, that is, a violation of the martingale restriction may be largely due to arbitrage activity. In other words, the empirical evidence suggests that the KOSPI 200 options market is not efficient. It is well known that many domestic individual investors with little trading experience and knowledge habitually buy KOSPI 200 options for simple and speculative reasons. In particular, there are active buyers of OTM and deep-OTM options, although these options are very unlikely to provide positive payoffs (see Kim & Ryu, 2012). Individuals buy these "cheap" options as if they are buying a lottery ticket or betting at a casino.[14] Hence, it is reasonable to expect that under these situations, sophisticated investors (e.g., foreign institutions) can take advantage of the differences between option-implied and market-observed spot prices to make arbitrage profits if they carefully monitor the price movements of options and index.

5. ROBUSTNESS CHECK

In this section we support the conclusions in section 4 by performing robustness checks. We first use futures as alternative underlying assets, and then divide the data sample into two periods,

[14] These views are also based on opinions of options market experts in Korea's leading financial institutions. In this regard, the authors are grateful to an executive director of SK Research Institute and a vice president of a leading bank.

before and after June, 2007, when the financial crisis began. Finally, we examine the accuracy of all models for KOSPI 200 option pricing.

5.1 Alternative Underlying Assets

Our test using KOSPI 200 index prices may have some limitations. First, we assume that investors can forecast future dividends perfectly until the maturity of each option contract, but this assumption may not reflect the reality.[15] Second, KOSPI 200 index prices may not reflect the fundamental value of the underlying asset because of problems related to non-synchronous trading and stale prices, which often arise when index prices are used.

We conduct the same empirical tests by using KOSPI 200 futures contracts to address these issues. KOSPI 200 futures contracts are very actively traded and the options and futures markets have the same closing time. In practice, traders often use cheaper futures contracts instead of the index itself to hedge index options. Therefore, this type of robustness check is widely employed (e.g., Longstaff, 1995; Brenner & Eom, 1997; Strong & Xu, 1999).

Although KOSPI 200 options and futures share many features, their maturity dates can be different. The maturity months for KOSPI 200 futures are March, June, September, and December, and thus, we select only those trading days on which options and futures contracts have the same nearest maturity dates. Table 8 reports the results for KOSPI 200 index futures. Largely consistent with the test using index values, all methods except the Brenner and Eom reject the null hypothesis for call options, and all reject it for put options at the 99% level of significance. The Longstaff model rejects the null hypothesis at the 99% level of significance for puts and provides positive price percentage differences (compared with the less significant differences in Table 4).

Similar results are obtained when we regress the percentage differences between implied and observed index futures prices on the variables for market friction. Table 9 presents the coefficient estimates and p-values[16]. The bid-ask adjusted spread becomes less significant, the total open interest, total trading volume and number of options have similar significance levels as for the underlying index. The explanatory power (R^2 value) changes only slightly, for example from 20.15% to 22.39% for calls and from 12.52% to 19.94% for puts for the BS model.

5.2 Sub-period Analysis

To investigate whether our findings are heavily biased on the exceptional market turbulence due to the recent financial crisis[17], we divide the sample periods into two sub-samples, one from January

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[15] Nevertheless, this assumption may have negligible effects because the time to maturity for options is typically less than a month and the average value of present dividends is much lower than the level of the KOSPI 200 index price.

[16] We also conduct a full regression using index futures values controlling additional variables as in Table 8. The results are similar, so we omit them. The results are available upon request.

[17] We thank an anonymous referee for raising this concern.

2002 to May 2007, and the other one from June 2007 to December 2010, as June 2007 is acknowledged as the start of the financial crisis. Then we re-conduct all tests using the two sub-samples separately.

Table 10 reports the results of percentage price differences between option-implied and observed values of the underlying index for two sub-samples[18]. All models except the Shimko model reject the null hypothesis at the 95% level of significance for the first sub-sample, and all models except the Longstaff model reject it at the 99% level of significance for the second sub-sample. The Brenner and Eom model returns similar *t*-statistics for the two sub-samples, the BS model has a larger *t*-statistic (thus rejects it more significantly) for the second sub-sample than the first sub-sample, while the Kernel regression has a larger *t*-statistic for the first sub-sample. The distinct *t*-statistics across samples may be attributed to the model characteristics. For instance, skewness and kurtosis are more pronounced in a volatile period than in a stable one, and, as a result, the distance between option-implied values and the underlying index values depends on how well the model can capture these characteristics in the distribution of the underlying asset.

5.3 Option Pricing Performance

We further examine the option pricing accuracy of all five models in this section, considering the importance of an accurate valuation for the validity of conclusions. Table 11 reports the statistics for pricing errors, defined as the differences between observed market prices and model prices of options. Not surprisingly, the BS model has the largest pricing error with a mean -0.0119 and a median -0.0124 for calls and a mean 0.0338 and a median 0.0399 for puts, compared with the smallest pricing error achieved by the Longstaff model for calls (mean=0.0021, median=0.0016), and by the Shimko model for puts (mean=0.0014, median=0.0008). These small pricing errors confirm that the models used are highly accurate and that violations of the martingale restriction are not likely due to model biases.

Overall, we find that the violation of the martingale restriction is robust to using index futures as an alternative underlying asset, to stable and volatile market conditions, and to model biases.

6. CONCLUSION

In this study, we test the martingale restriction for the KOSPI 200 options market. To avoid any model misspecification, we use both parametric and nonparametric methods to estimate the risk-neutral density function. We then employ this function to compute option-implied index prices. Percentage differences between option-implied and market-observed values and *t*-test results suggest a violation of the martingale restriction for the market for most of the methods employed.

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[18] We find qualitatively the same regression results on market friction variables for the two sub-samples as for the whole sample, results are available upon request.

The regression analysis indicates that the variables for market friction, including bid-ask adjusted spread, total trading volume, open interest, and the number of options, account for only a small portion of the percentage differences. After considering several alternative explanations, we conclude that the Korean options market, unlike its U.S. counterpart, has substantial arbitrage opportunities and is thus not efficient.

Our conclusions are robust. First, we use both parametric and nonparametric methods to test the martingale restriction, whereas previous studies consider only parametric methods. Second, we divide the data sample into two sub-samples to avoid possible bias due to market condition. Third, we replace index values by index futures values to avoid problems related to non-synchronous trading and stale prices. Further, for the regression analyses, we control for time-to-maturity, moneyness, and volatility biases to examine the percentage differences between implied and observed underlying values. Our finding of the inefficiency of the KOSPI 200 options market has critical implications for both investors and regulator.

REFERENCES

Ahn, H. J., Kang, J., & Ryu, D. (2008). Informed trading in the index option market: The case of KOSPI 200 options. Journal of Futures Markets, 28, 1118-1146.

Ahn, H. J., Kang, J., & Ryu, D. (2010). Information effects of trade size and trade direction: Evidence from the KOSPI 200 index options market. Asia-Pacific Journal of Financial Studies 39, 301-339.

Ait-Sahalia, Y., & Lo, A. W. (1998). Nonparametric estimation of state-price densities implicit in financial asset prices. Journal of Finance, 53, 499-547.

Bahra, B. (1997). Implied risk-neutral probability density functions from option prices: theory and application. Bank of England Working Paper No. 66.

Breeden, D. T., & Litzenberger, R. H. (1978). Prices of state-contingent claims implicit in option prices. Journal of Business, 51, 621-651.

Brenner, M., & Eom, Y. H. (1997). No-arbitrage option pricing: new evidence on the validity of the martingale property. NYU Working Paper No. FIN-98-009.

Figlewski, S. (2008). Estimating the implied risk neutral density for the U.S. market portfolio. Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle.

Harrison, J. M., & Kreps, D. M. (1979). Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory, 20, 381-408.

Jackwerth, J. C. (1999). Option implied risk-neutral distributions and implied binomial trees: A literature review. Journal of Derivatives, 7, 66-82.

Kim, H., & Ryu, D. (2012). Which trader's order-splitting strategy is effective? The case of an index options market. Applied Economics Letters, forthcoming

Li, H., & Zhao, F. (2009). "Nonparametric Estimation of State-Price Densities Implicit in Interest Rate Cap Prices." Review of Financial Studies 22, 4335-4376.

Longstaff, F. A. (1995). Option pricing and the martingale restriction. Review of Financial Studies, 8, 1091-1124.

Manaster, S., & Rendleman, J. R. J. (1982). Option prices as predictors of equilibrium stock prices. Journal of Finance, 37, 1043-1057.

Merton, R. C. (1973). Theory of rational option pricing. Bell Journal of Economics, 4, 141-183.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics, 3, 125-144.

Neumann, M., & Schlag, C. (1996). Martingale restrictions and implied distributions for German stock index option prices. Working Paper, University at Karlsruhe.

Ryu, D. (2011). Intraday price formation and bid/ask spreads: A cross market approach. Journal of Futures Markets, 31, 1142-1169.

Shimko, D. (1993). Bounds of probability. Risk Magazine, 6(4).

Stone, C. J. (1984). An asymptotically optimal window selection rule for kernel density estimates. Annals of Statistics 12, 1285-1297.

Strong, N., & Xu, X. (1999). Do S&P 500 index options violate the martingale restriction? Journal of Futures Markets, 19, 499-521.

Turvey, C. G., & Komar, S. (2006). Martingale restrictions and the implied market price of risk. Canadian Journal of Agricultural Economics, 54, 379-399.

Uller, Marlene M. (1997). Multivariate and semiparametric kernel regression. MG Schimek (Ed.), Smoothing and Regression. Approaches, Computation and Application, Wiley.

Table 1

Top 10 Index Futures and Options Worldwide

Rank	Contract	Index Multiplier	2010
1	KOSPI200 options, KRX	KRW 100,000	3,525,898,562
2	E-mini S&P 500 index futures, CME	USD 50	555,328,670
3	SPDR S&P 500 ETF options, multiple exchanges	-	456,863,881
4	S&P CNX Nifty index options, NSE (India)	INR 100	529,773,463
5	Euro Stoxx50 futures, Eurex	EUR 10	372,229,766
6	Euro Stoxx50 index options, Eurex	EUR 10	284,707,318
7	RTS index futures, RTS	USD 2	224,696,733
8	S&P 500 index options, CBOE	USD 100	175,291,508
9	S&P CNX Nifty index futures, NSE (India)	INR 100	156,351,505
10	Nikkei225 Mini futures, OSE	JPY 100	125,113,769

Note. This table shows the 10 most active derivatives contracts for index futures and options. The contracts are ranked by the number of contracts traded and/or cleared in 2010. The SPDR S&P 500 ETF options are traded on multiple U.S. options exchanges. Source: *Futures Industry Association* (www.futuresindustry.org).

Table 2

Trading Activity by Investor Types

Investor Group	Total (in contracts)	Percentage (%)
Domestic individuals	20,400,597,212	42.3
Domestic institutions	18,065,933,152	37.4
Foreigners	9,777,346,354	20.3
Total	48,243,876,718	100.0

Note. The table presents summary statistics for trading activity in the KOSPI200 options market by investor type (domestic individuals, domestic institutions, and foreigner investors) for the period between January 2002 and December 2010. Trading activity is indicated by the number of contracts.

Table 3
Summary Statistics for Options

Panel A. KOSPI200 index

	Mean	Std. Dev.	Min	Median	Max
Index value	160.58	54.27	65.58	168.23	271.19

Panel B. Call options

	Mean	Std. Dev.	Min	Median	Max
Number	9.97	3.75	1	9	20
Moneyness	-0.618	2.364	-11.160	-0.540	13.280
Time to maturity	0.0423	0.0242	0.0000	0.0407	0.0984

Panel C. Put options

	Mean	Std. Dev.	Min	Median	Max
Number	10.68	3.74	1	10	20
Moneyness	1.344	2.501	-12.967	0.810	11.690
Time to maturity	0.0423	0.0242	0.0000	0.0407	0.0984

Note. This table shows daily summary statistics for KOSPI200 index and for KOSPI200 call and put options for the period between January 2002 and December 2010. "*Index value*" the KOSPI200 index price. "*Number*" denotes the number of call options, "*Moneyness*" for call options the average difference between their index value and strike price for a given day, "*Moneyness*" for put options the average difference between their strike price and index value for a given day, and "*Time to maturity*" the (annualized) average time to maturity for a given day.

Table 4

Percentage Price Differences between Option-Implied and Observed Values of the Underlying Index

Panel A. Call options

	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.1409	0.3235	-0.6829	0.1192	1.7932	18.1990
Longstaff	-0.0583	0.4281	-1.5143	-0.0005	1.3643	-5.6983
Brenner & Eom	-0.5622	2.2830	-13.0110	-0.5856	7.8536	-10.3180
Non-Parametric						
Shimko	0.0318	0.1666	-0.6113	0.0366	0.6392	7.9278
Kernel Regression	1.2908	0.9253	-2.3719	1.3101	3.6328	58.4900

Panel B. Put options

	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.2055	0.5226	-0.7114	0.0902	2.7967	16.7660
Longstaff	-0.0315	0.5842	-1.6151	-0.0942	2.7712	-2.3032
Brenner & Eom	-1.8905	4.9912	-25.3970	0.2562	2.3973	-16.1900
Non-Parametric						
Shimko	0.0292	0.1634	-0.5722	0.0336	0.6411	7.5911
Kernel Regression	-1.1832	0.9630	-3.7124	-1.1878	2.0265	-52.4750

Note. This table presents the summary statistics for percentage differences between option-implied and market-observed values of the underlying index for call and put options. Here *t*-statistics are used for the null hypothesis H_0 that there is no difference. The critical values are 1.646, 1.962, and 2.580 for the 10%, 5%, and 1% levels of significance, respectively, for the two-tailed *t*-test.

Table 5

Estimates of the Bandwidth *h* in Kernel Regressions

Panel A. Call options

	Index value	Strike price	Time to maturity	Risk free rate (%)	R^2
Kernel Regression	0.6084	1.1597	0.0088	0.0005	0.9881

Panel B. Put options

	Index value	Strike price	Time to maturity	Risk free rate	R^2
Kernel Regression	0.5947	1.2006	0.0085	0.0004	0.9779

Note. This table shows the estimates of the bandwidth *h*, which are obtained by minimizing the least squares cross-validation. The kernel regression (KR) method is four-dimensional with option price as the dependent variable. R^2 is the adjusted *R*-squared value.

Table 6

Regression Analysis using KOSPI200 Index Values

Panel A. Call options

	C	*BA*	*OI*	*TTV*	*N*	R^2
Parametric						
BS	-0.1808***	-0.6994	-0.9504***	0.0016***	0.0360***	0.2015
P-value	*0.0000*	*0.1260*	*0.0001*	*0.0047*	*0.0000*	
Longstaff	-0.2490***	1.0790	0.1101	-0.0004	0.0154***	0.0138
P-value	*0.0000*	*0.1073*	*0.7557*	*0.6448*	*0.0000*	
Brenner & Eom	-0.5315**	-15.3850***	8.9319***	-0.0098**	-0.0077	0.0120
P-value	*0.0114*	*0.0000*	*0.0000*	*0.0257*	*0.6520*	
Non-Parametric						
Shimko	-0.0469***	-0.4470*	-0.3707***	0.0006*	0.0101***	0.0667
P-value	*0.0017*	*0.0874*	*0.0054*	*0.0694*	*0.0000*	
Kernel Regression	2.9432***	-3.2882***	-3.3692***	-0.0003	-0.1179***	0.2459
P-value	*0.0000*	*0.0098*	*0.0000*	*0.8660*	*0.0000*	

Panel B. Put options

	C	*BA*	*OI*	*TTV*	*N*	R^2
Parametric						
BS	0.1256***	5.2247***	-6.0406***	0.0039***	0.0293***	0.1252
P-value	*0.0063*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	
Longstaff	0.0989*	3.0889***	-0.6989	0.0020**	-0.0197***	0.0219
P-value	*0.0666*	*0.0005*	*0.1603*	*0.0322*	*0.0000*	
Brenner & Eom	-3.7303***	23.5540***	-6.2187	-0.0137	0.1976***	0.0140
P-value	*0.0000*	*0.0019*	*0.1498*	*0.1023*	*0.0000*	
Non-Parametric						
Shimko	-0.0048	0.0780	-0.7832***	0.0003	0.0079***	0.0491
P-value	*0.7481*	*0.7602*	*0.0000*	*0.3071*	*0.0000*	
Kernel Regression	-3.3364***	8.6515***	2.2135***	0.0058***	0.1388***	0.3640
P-value	*0.0000*	*0.0000*	*0.0008*	*0.0000*	*0.0000*	

Note. This table shows the coefficient estimates and *p*-values obtained from the regression of percentage differences between option-implied index values and market-observed index values on variables for market friction. Here *C* denotes the regression intercept, *BA* the bid-ask adjusted spread, *OI* the open interest, *TTV* the total trading volume, and N the number of call/put options for a given day. The coefficients for *OI* and *TTV* are multiplied by 10^7 and 10^{10} respectively; R^2, adjusted *R*-squared value for the regression.

Table 7

Full Regression Analysis using KOSPI200 Index values

Panel A. Call options

	C	*T*	*M*	*BA*	*OI*	*TTV*	*N*	*ARet*	*ARet-1*	*ARet-2*	R^2
Parametric											
BS	-0.2639***	-1.1502***	0.0247***	0.0480	-0.5633**	0.0003	0.0420***	1.4456***	2.6053***	2.1933***	0.2521
P-value	*0.0000*	*0.0060*	*0.0000*	*0.9269*	*0.0294*	*0.5777*	*0.0000*	*0.0060*	*0.0000*	*0.0000*	
Longstaff	-0.1687***	-2.9963***	0.0426***	2.0964***	-0.2434	-0.0015*	0.0243***	1.0770	1.4171*	1.0303	0.0751
P-value	*0.0081*	*0.0000*	*0.0000*	*0.0065*	*0.5211*	*0.0900*	*0.0000*	*0.1769*	*0.0702*	*0.1858*	
Brenner & Eom	-0.1115	-2.3152	0.1519***	-8.3229*	4.7917**	-0.0052	-0.0034	-8.3631*	-4.5511	-3.3170	0.0313
P-value	*0.7488*	*0.4855*	*0.0000*	*0.0537*	*0.0220*	*0.2735*	*0.8523*	*0.0589*	*0.2913*	*0.4381*	
Non-Parametric											
Shimko	-0.0761***	-0.2902	-0.0007	-0.6096**	-0.1312	0.0001	0.0115***	0.6486**	0.7011**	0.7076**	0.0740
P-value	*0.0023*	*0.2234*	*0.7171*	*0.0458*	*0.3781*	*0.8548*	*0.0000*	*0.0351*	*0.0207*	*0.0180*	
Kernel Regression	2.6801***	3.7510***	-0.1763***	-10.7520***	-0.2292	-0.0029*	-0.1313***	-1.0596	4.1431***	4.3685***	0.4043
P-value	*0.0000*	*0.0004*	*0.0000*	*0.0000*	*0.7272*	*0.0542*	*0.0000*	*0.4426*	*0.0022*	*0.0011*	

Panel B. Put options

	C	*T*	*M*	*BA*	*OI*	*TTV*	*N*	*ARet*	*ARet-1*	*ARet-2*	R^2
Parametric											
BS	-0.0395	1.8903***	-0.1012***	3.0611***	-5.9941***	0.0044***	0.0110***	4.3493***	4.5522***	3.6823***	0.2967
P-value	*0.5667*	*0.0037*	*0.0000*	*0.0001*	*0.0000*	*0.0000*	*0.0033*	*0.0000*	*0.0000*	*0.0000*	
Longstaff	0.2545***	0.7160	-0.0649***	1.0804	-1.6748***	0.0039***	-0.0372***	-1.1559	0.9310	-1.5410	0.0747
P-value	*0.0038*	*0.3881*	*0.0000*	*0.2635*	*0.0018*	*0.0001*	*0.0000*	*0.3021*	*0.3897*	*0.1558*	
Brenner & Eom	-4.1474***	16.5280**	-0.2981***	21.0870**	-8.0737*	-0.0024	0.1070**	-11.5500	7.8706	6.9116	0.0305
P-value	*0.0000*	*0.0234*	*0.0000*	*0.0130*	*0.0857*	*0.7799*	*0.0106*	*0.2471*	*0.4150*	*0.4775*	
Non-Parametric											
Shimko	-0.0416*	0.0758	-0.0053***	-0.0179	-0.6690***	0.0001	0.0077***	0.6381**	0.5175*	0.7503**	0.0576
P-value	*0.0967*	*0.7471*	*0.0046*	*0.9501*	*0.0000*	*0.6484*	*0.0000*	*0.0411*	*0.0889*	*0.0138*	
Kernel Regression	-3.2506***	-6.4875***	0.1640***	11.2880***	3.4124***	0.0008	0.1853***	1.5619	-3.0887**	-2.8447**	0.4955
P-value	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.4822*	*0.0000*	*0.2688*	*0.0229*	*0.0373*	

Note. This table shows the coefficients estimates and *p*-values obtained from the regression of percentage differences between option-implied index values and market-observed index values on selected independent variables. Panel A shows the results for call options, and Panel B, those for put options. Here *C* denotes the regression intercept, *T* the average time to maturity, *M* the average moneyness of all call/put options for a given day, *BA* the bid-ask adjusted spread, *OI* the open interest, *TTV* the total trading volume, *N* the number of call/put options for a given day, and *ARet*, *ARet-1*, and *ARet-2* are the current, first lagged, and second lagged values of absolute daily returns on the KOSPI200 index. The coefficients for *OI* and *TTV* are multiplied by 10^7 and 10^{10}, respectively; R^2, adjusted *R*-squared value for the regression.

Table 8

Percentage Differences between Option-Implied and Market-Observed Index Futures Prices

Panel A. Call options

	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.2720	0.3201	-0.3854	0.2122	1.7336	20.2160
Longstaff	0.0959	0.4499	-1.3135	0.0591	1.6601	5.0728
Brenner & Eom	-0.1055	2.2273	-7.6642	-0.3868	9.7409	-1.1289
Non-Parametric						
Shimko	0.0527	0.1699	-0.5797	0.0593	0.6452	7.3443
Kernel Regression	-1.1949	1.3056	-3.7516	-1.3131	1.9559	-21.7530

Panel B. Put options

	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.3445	0.4591	-0.3358	0.2322	2.8555	18.3030
Longstaff	0.0914	0.5794	-1.2332	0.0099	3.0468	3.8547
Brenner & Eom	-1.8687	5.4333	-27.1480	0.4811	2.3210	-8.4176
Non-Parametric						
Shimko	0.0511	0.1715	-0.5682	0.0621	0.6510	7.2441
Kernel Regression	-0.9344	1.1253	-3.8418	-0.9773	2.7370	-20.2890

Note. This table shows the summary statistics for percentage differences between option-implied and market-observed futures prices for call and put options. Here *t*-statistics are used for the null hypothesis H_0 that there is no difference. The critical values are 1.6487, 1.9659, and 2.5882 for the 10%, 5%, and 1% levels of significance for the two-side *t*-test.

Table 9

Regression Analysis using KOSPI200 Index Futures Values

Panel A. Call options

	C	*BA*	*OI*	*TTV*	*N*	R^2
Parametric						
BS	0.2349***	1.1877	-3.8107***	0.0031***	0.0239***	0.2239
P-value	*0.0000*	*0.1309*	*0.0000*	*0.0004*	*0.0000*	
Longstaff	0.2691***	2.8768**	-2.5459***	0.0003	-0.0025	0.0314
P-value	*0.0002*	*0.0202*	*0.0001*	*0.8541*	*0.6592*	
Brenner & Eom	-0.5240	-2.3327	1.9411	0.0030	0.0192	-0.0026
P-value	*0.1375*	*0.7101*	*0.5515*	*0.6729*	*0.4999*	
Non-Parametric						
Shimko	0.0271	0.1700	-1.2044***	0.0008	0.0098***	0.0968
P-value	*0.2900*	*0.7143*	*0.0000*	*0.1235*	*0.0000*	
Kernel Regression	0.9559***	19.2750***	-16.9590***	0.0203***	-0.1711***	0.3593
P-value	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	

Panel B. Put options

	C	*BA*	*OI*	*TTV*	*N*	R^2
Parametric						
BS	0.2658***	2.1809**	-5.6957***	0.0040***	0.0337***	0.1994
P-value	*0.0001*	*0.0474*	*0.0000*	*0.0005*	*0.0000*	
Longstaff	0.4113***	-1.6660	0.3410	0.0009	-0.0298***	0.0221
P-value	*0.0000*	*0.2764*	*0.7189*	*0.5790*	*0.0001*	
Brenner & Eom	-5.5717***	12.8890	17.1130**	-0.0518***	0.3241***	0.0415
P-value	*0.0000*	*0.3658*	*0.0493*	*0.0006*	*0.0000*	
Non-Parametric						
Shimko	0.0044	0.0171	-1.1473***	0.0010**	0.0098***	0.0981
P-value	*0.8662*	*0.9695*	*0.0000*	*0.0332*	*0.0000*	
Kernel Regression	-3.2236***	2.6033	2.2452	0.0066***	0.1606***	0.3858
P-value	*0.0000*	*0.2708*	*0.1238*	*0.0097*	*0.0000*	

Note. This table shows the coefficients estimates and *t*-statistics obtained from the regression of percentage differences between option-implied values of index futures and market-observed prices of index futures on variables for market friction. Here *C* denotes the regression intercept, *BA* the bid-ask adjusted spread, *OI* the open interest, *TTV* the total trading volume, and *N* the number of call/put options for a given day. The coefficients for *OI* and *TTV* are multiplied by 10^7 and 10^{10}, respectively; R^2, adjusted *R*-squared value for the regression.

Table 10

Percentage Price Differences between Option-Implied and Observed Values of the Underlying Index for Two Sub-Samples

Panel A. First sample (Jan, 2002 ~ May, 2007)

Call options	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.0231	0.2526	-0.8368	0.0492	0.8306	2.9819
Longstaff	-0.0996	0.4420	-1.5982	-0.0048	1.1162	-7.3479
Brenner & Eom	-0.4423	2.1079	-9.6619	-0.5794	8.3980	-6.8477
Non-Parametric						
Shimko	-0.0033	0.2156	-0.7209	0.0085	0.7503	-0.5001
Kernel Regression	1.5201	0.7351	-0.8620	1.4991	3.4769	67.4850

Put options	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.0222	0.3334	-0.8565	0.0063	1.2173	2.2192
Longstaff	-0.0581	0.5691	-1.6514	-0.1053	1.7564	-3.3982
Brenner & Eom	-2.3695	5.4523	-28.5350	-0.1741	1.9371	-14.4980
Non-Parametric						
Shimko	-0.0030	0.2134	-0.7692	0.0129	0.7912	-0.4695
Kernel Regression	-1.5104	0.8559	-4.1656	-1.3975	0.8288	-58.9260

Panel B. Second sample (June, 2007 ~ Dec, 2010)

Call options	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.3241	0.3836	-0.3337	0.2404	2.2616	22.1260
Longstaff	-0.0052	0.4142	-1.0470	0.0122	1.8061	-0.3314
Brenner & Eom	-0.7056	2.4943	-14.5370	-0.5821	7.8536	-7.4306
Non-Parametric						
Shimko	0.0769	0.0936	-0.1769	0.0685	0.5457	21.5190
Kernel Regression	0.9231	1.0844	-2.6118	0.9625	3.8246	22.3770

Put options	Mean	Std.dev	Min	Median	Max	*t*-stat.
Parametric						
BS	0.4915	0.7082	-0.4576	0.2943	4.4202	18.5420
Longstaff	0.0123	0.6540	-1.6081	-0.0775	4.0529	0.5016
Brenner & Eom	-1.2338	4.3331	-24.6650	0.6052	2.8872	-7.6033
Non-Parametric						
Shimko	0.0729	0.0932	-0.1777	0.0629	0.4960	20.8420
Kernel Regression	-0.7091	0.9813	-2.9215	-0.8187	2.6366	-19.2950

Note. This table presents the summary statistics for percentage differences between option-implied and market-observed values of the underlying index for call and put options for two sub-samples; one is from January, 2002 to May, 2007, the other one is from June, 2007 to December, 2010. Here *t*-statistics are used for the null hypothesis H_0 that there is no difference. The critical values are 1.646, 1.962, and 2.580 for the 10%, 5%, and 1% levels of significance, respectively, for the two-tailed *t*-test.

Table 11

Option Pricing Performance Comparison

Panel A. Call options

	Mean	Std.dev	Min	Median	Max
Parametric					
BS	-0.0119	0.1419	-2.4221	-0.0124	2.5938
Longstaff	0.0021	0.1050	-1.9351	0.0016	2.7289
Brenner & Eom	-0.0121	0.3652	-3.8024	-0.0029	3.9965
Non-Parametric					
Shimko	0.0032	0.1188	-1.1104	-0.0010	3.5288
Kernel Regression	0.0116	0.2075	-1.6783	-0.0079	1.7783

Panel B. Put options

	Mean	Std.dev	Min	Median	Max
Parametric					
BS	0.0338	0.1662	-1.6974	0.0399	2.9244
Longstaff	0.0024	0.0981	-1.7752	0.0022	2.6353
Brenner & Eom	-0.0227	0.2189	-2.9814	-0.0027	3.9683
Non-Parametric					
Shimko	0.0014	0.1168	-1.7983	0.0008	3.0643
Kernel Regression	0.0133	0.2233	-1.4474	-0.0082	1.9106

Note. This table shows the statistics of option pricing errors for all five models, the whole data sample is used, where pricing errors are defined as the differences between observed market prices and model prices of options.