The Role of Variability and Prior Knowledge in Learning from Comparing Multiple Examples*

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Abstract Researchers have consistently shown that studying multiple examples is more effective than one example to promote learning because the comparison evoked by comparing multiple examples is generally good for learning. Not all comparisons, however, may equally be effective. Principles found in the literature for designing multiple examples remain ambiguous. This paper reviews experimental studies on learning from comparison of examples and identifies issues that have not been resolved: (a) how similar or different examples should be to facilitate learning, and (b) the role of students' prior knowledge in example - based learning. The review addresses these limitations and provides recommendations according to our empirical studies. First, example design should focus on aspects and features that are critical for student learning, and use the distinction of critical/uncritical instead of surface/structural. Second, students may need to separately discern each critical aspect before they can benefit from comparing simultaneous variation of these aspects. Third, students with different levels of prior knowledge may perceive different aspects of examples as critical for their learning. Examples should be designed according to aspects that are critical to specific students. Suggestions for future research are provided.

Key words multiple examples, variability, prior knowledge, comparison

Using examples is a useful and widespread instructional technique adopted by teachers; learning from examples is a very effective method of cognitive skill development and has been a major topic in educational research for at least the past four decades (Atkinson, Derry, Renkl, & Wortham, 2000; Renkl, 2014). There is an enormous body of cognitive literature that examines the importance of examples and provides empirical support for principles of designing examples that are conducive to learning (e. g., Namy & Gentner, 2002; Schwartz & Bransford, 1998).

Learning from examples is complex, and its effectiveness could be moderated by several factors. A great deal has been learned about example variability and its affecting factors, and a great deal remains to be learned. In view of this, our intention in this review is not to demonstrate all potential factors in the effectiveness of learning from comparing multiple examples. Rather, we focus on one known factor, example variability. We first review literature that argues multiple examples are better than one for learning and explains the reasoning. We then elaborate on the effect of the example variability on comparison and learning, followed by a discussion of the effect of students' prior knowledge on comparison and learning. Finally, we highlight the limitations of existing cognitive research on learning from comparison of examples and present our recommendations for future research, according to our empirical studies.

1 Quantity of examples: Multiple examples or one example?

Among these issues regarding how to design best examples for learning, the quantity of examples is one of the problems that have been intensively investigated by researchers. The question is: how many examples

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are needed for learning?

Plenty of studies on learning from examples have shown that multiple examples are better than one in terms of promoting learning (e. g. , Gentner, 2005; Schwartz & Bransford, 1998). Researchers generally agree that comparison evoked by multiple examples is good for learning. As Gentner and Namy (1999) argued, comparison among examples can highlight common structure which is important for learning.

Comparing multiple examples was found to be more effective than one single example for learning because comparison leads to schema abstraction and formation (Reed & Bolstad, 1991). Tennyson and colleagues (Merril & Tennyson, 1978; Tennyson, 1973) have demonstrated that comparing multiple examples including positive and negative examples is necessary for concept learning. As Tennyson (1973) argued, comparing negative examples with positive examples helps the learner concentrate on the relevant attribute of a concept to be learned; without the use of negative examples, the learner might consider an irrelevant attribute as a relevant attribute. In addition, various positive examples are needed to generalize the learning of the relevant attribute.

Researchers also demonstrated the effectiveness of comparing multiple examples for problem solving. Gick and Holyoak (1983) conducted one of the earliest studies of the effectiveness of comparison on learning to solve a difficult problem. In their study, college students were given the Duncker radiation problem to solve. Prior to solving the problem, some students read a story with solution that was analogically similar to the radiation problem, but the other students read two analogically similar stories with solutions and compared the commonalities between the two stories. Results showed that students who read and compared the two stories performed much better in solving the later Duncker radiation problem than did those who read one story only.

Gick and Holyaok (1983) concluded that two analogically similar problems can be mapped together to induce a more general problem schema which can be retrieved for later new problem solving, and thus facilitate subsequent transfer. As they argued, a schema should exist in memory before solving a problem, and the schema could only be achieved by comparing multiple related examples. The schema is easily retrieved and mapped because it is abstract, general, and more similar to the target problem. In contrast, if students are given one single example only, when they attempt to solve a problem, it would be more difficult for them to retrieve and map the prior example because of the differences between the example and the target problem.

Gentner and colleagues (Gentner & Namy, 1999; Loewenstein, Thompson, & Gentner, 2003; Namy & Gentner, 2002) carried the question a step further by proposing the structure - mapping theory of comparison. According to the theory, comparison is not the process of similarity - based abstraction resulting in a general representation with some relevant attributes, but the process of structural alignment. When multiple examples are compared, common structure of the examples that may not have been explicitly assessed prior to comparison is aligned and highlighted, which is crucial for deeper learning.

Schwartz and his colleagues (Schwartz & Bransford, 1998; Schwartz & Martin, 2004) also demonstrated the effectiveness of comparing multiple examples on learning, but interpreted the comparison from a different perspective. As they argued, comparing multiple examples can help learners generate the differentiated knowledge structures that promote subsequent learning. In particular, Schwartz and associates interpreted the process of comparison as discernment. Discerning important features and structures of examples "helped the students discover the characteristic features of the target phenomena" and "the features could serve as a set of cues that reminded the students of the target concepts" (Schwartz & Bransford, 1998).

To sum up, comparing multiple examples is generally suggested for learning, because the comparison generated from multiple examples can help to form a schema, to align the common structure, or to discern the important features of examples, which promotes learning. Not all comparison, however, may be equally effective (Rittle - Johnson, Star, & Durkin, 2009); different sets of multiple examples can lead to different kinds of comparison that might have different impacts

on learning. So the question now becomes: how to design multiple examples to develop effective comparison for learning?

2 Variability of multiple examples: "Similar" or "different"?

The effectiveness of comparing multiple examples depends on the variability of examples being compared, i. e., how similar or different the examples are (Gentner & Namy, 1999; Renkl, Stark, Gruber, & Mandl, 1998; Rittle - Johnson & Star, 2009). Learners might discern different aspects of examples and develop different knowledge regarding a phenomenon when they compare different kinds of examples.

Examples are generally analyzed from surface (irrelevant) features and structural (relevant) features. Example variability consists of differences in the surface and structural aspects among the examples. Holyoak and Koh (1987) distinguished surface features, such as names, objects, numbers, and story lines, as irrelevant to goal attainment and structural features, such as underlying mathematical procedures, rules, solutions, and principles, as relevant to goal attainment.

It remains unclear how similar or different examples should be in order to promote learning (Renkl et al., 1998; Rittle - Johnson & Star, 2009). In the case of examples with very dissimilar surface features, it is often more difficult for students to find the underlying common structure (Richland, Holyoak, & Stigler, 2004; Ross, 1989). Conversely, when given examples that have strong surface similarity, students may wrongly identify surface features as crucial (Quilici & Mayer, 1996).

2.1 Example variability for concept learning

Tennyson and his colleagues (Merrill & Tennyson, 1978; Tennyson, 1973) conducted empirical studies examining the role of positive and negative examples in concept learning. In particular, they proposed a rational set of positive and negative examples for learning in which divergent positive examples were presented with matched negative examples and arranged from easy to difficult. Matched refers to the relation between a positive example and a negative example. Two examples are matched if they have the same surface features and are different in their structural features. Divergent refers to the relation between positive examples. Positive examples are divergent if they are different in their surface features. Difficulty refers to how difficult positive examples and negative examples are.

Similar findings were reported by Ranzijin (1991) who found that positive examples of a concept presented to students should be divergent in surface features to develop a more accurate understanding of the concept. Hammer, Bar - Hillel, Hertz, Weinshall, and Hochstein (2008) also demonstrated that surface features of positive examples should differ in number and distance in order to make the example comparison more informative. For a pair of positive and negative examples to more effectively support learning, the difference in their surface features should be small in number and distance and be smaller than the difference in their structural features.

In contrast, Gentner and her colleagues did not find that positive examples should be as different as possible in terms of surface features to facilitate learning. Instead, they argued that common surface features among examples can initially prompt the structure mapping and alignment process and "may invite the child to notice further, more abstract, commonalities" (Gentner & Namy, 1999).

Namy and Gentner (2002) conducted a study to investigate the effects of superficial similarity and structural similarity on structure alignment and learning. In the study, children in the one - kind condition compared two superficially similar examples of a taxonomic category; children in the two - kind condition compared two superficially similar objects from two different categories. Then the children were given two choice alternatives: the category choice and the perceptual choice. Results indicated that children in the one - kind condition were more likely to select the category choice than those in the two - kind condition. The authors thus concluded that the superficially similar examples to be compared should also be structurally similar for children to yield category - based responding. Conversely, if the examples are only similar in surface features rather than in structural features, children would be more likely to focus on the surface features.

Namy and Clepper (2010) carried the findings a step further to examine the effects of comparison and contrast on categorization in four conditions: No - Compare/No - Contrast, Compare/No - Contrast, No - Compare/Contrast, and Compare/Contrast. The No - Compare/No - Contrast condition contained one single example from the target category. The Compare/No - Contrast condition and the No - Compare/Contrast condition were the same as the one - kind condition and two - kind condition in Namy and Gentner's (2002) study. The Compare/Contrast condition included examples from the Compare/No - Contrast and the No - Compare/Contrast conditions: perceptually similar examples from the target category and from out of the target category. The findings were consistent with Namy and Gentner's (2002) study by showing that children in the Compare conditions displayed more taxonomic responding than did those in the Contrast conditions. In addition, Namy and Clepper (2010) found that contrast preceded by comparison facilitated taxonomic responding more than did comparison alone; "contrast apparently contributes to children's categorization only within the context of comparison".

These studies, however, did not support the argument that superficially similar examples are better than superficially different examples in promoting learning because the authors did not compare the effectiveness of one condition of superficially different examples and another condition of superficially similar examples. Furthermore, Namy and Clepper (2010) examined the different roles of comparison and contrast in taxonomic categorization and recommended that learners should compare examples from the target category rather than examples from out of the target category. The variability of positive and negative examples, however, was not investigated with respect to their relative effectiveness in promoting learning.

2.2 Example variability for procedural learning

Contradictory findings also have been reported in research on the effectiveness of using worked examples as instructional methods. After showing the advantage of multiple examples over no examples in learning three statistical principles in Experiment 1, Quilici and Mayer (1996) further investigated the effect of superficial variability of examples in Experiment 2. They divided surface features of examples into two aspects (surface story and value) and developed two example sets that differed in the two aspects: the surface - emphasizing example set used the same surface story with different values for all examples of each principle, whereas the structure - emphasizing example set used different surface stories and values for all examples of each principle.

College students were randomly assigned to the surface - emphasizing group or the structure - emphasizing group. After instruction, students were asked to complete statistical tasks. Results indicated that students studying the structure - emphasizing example set were more likely to sort subsequent problems on the basis of structure or apply appropriate statistical principles than those receiving the surface - emphasizing example set. The authors therefore recommended using examples with different surface stories to help students to focus on structural features rather than relying on surface features, which might promote construction of structure - based schema.

Similar findings were obtained by other researchers. In Paas and Van Merrienboer's (1994) study, students in a secondary technical school were randomly assigned to a high - variability examples group or a low - variability examples group to study solving a geometrical problem in the domain of computer numerically controlled machinery programming. The high - variability examples had different values and different problem formats, whereas the low - variability examples had different values only. Results showed that students who studied high - variability examples outperformed those who studied low - variability examples on the transfer posttest. The authors thus concluded that the high - variability examples promoted schema acquisition. Ning and Yu (2010) examined the effect of example variability in learning counting principle. They found that the high variable examples were significantly better than the low variable examples in helping students solve medium and far transfer problems.

To sum up, studies above suggested that comparing high - variability examples that differ in all surface features leads to better learning than comparing low -

variability examples that differ in value only. This is because high variability of examples increases germane cognitive load rather than extraneous cognitive load. Students comparing high – variability examples are more likely to focus on the structural features of the examples and construct structure – based schema, whereas students comparing low – variability examples are more likely to focus primarily on surface features. Sweller (2010) recently used element interactivity to account for the effect of example variability. As he claimed, high – variability examples have substantially higher element activity and intrinsic cognitive load than low – variability examples. Thus, in a learning situation with low extraneous load, high – variability examples would increase germane cognitive load, which refers to the working memory resources available to deal with the element interactivity associated with intrinsic cognitive load as well as with learning.

Conversely, other researchers have suggested that superficially similar examples should be used during instruction because they help students to pay attention to the structural aspects (Ross, 1989). For instance, Renkl et al. (1998) did not find the effects of high – variability examples to be more beneficial for learning. In their study, vocational college students were given uniform examples that had the same surface story or multiple examples that had different surface stories to study solving an economic problem about interest rate. Then, they assessed students' performance on a near transfer test and a far transfer test. Results did not indicate the advantage of high – variability multiple examples over uniform examples on either near or far transfer tests. The authors found that high – variability multiple examples led to poor learning without instructional support. They thus concluded that highly variable examples might overtax the students and were not recommended for instruction.

Those inconsistent findings suggested ambiguity regarding how similar multiple examples should be with respect to surface features. A possible explanation may lie in the learner's working memory capacity. As Sweller, van Merrienboer, and Paas (1998) argued, variability has a positive effect on learning only if the learning situation has low extraneous cognitive load and the total cognitive load does not overburden the learners' working memory; the effectiveness of highly variable examples reverses when the learners' working memory is overtaxed.

The studies described above did not investigate the effect of the variability of structural features on learning because examples in these studies differed in surface features but shared the same structural features or underlying principles. In contrast, Gick and Paterson (1992) examined the effects of both superficial similarity and structural similarity on learning to solve a difficult problem, which was an extension of Gick and Holyoak's (1983) study described previously. To further explore how to facilitate schema induction by comparing two examples, Gick and Paterson (1992) added a third example for students who had already read two "convergence" stories as in Gick and Holyoak's (1983) study. The additional example, however, had two different forms: the near miss condition and the unrelated condition. The near miss example (i. e. a negative example) had the same surface features as the two convergence stories, but had different structural features as it provided an ineffective sequential attack method instead of a simultaneous method; the unrelated example was different from the two convergence stories in terms of surface and structural features.

Results showed that students in the near miss condition outperformed those in the unrelated condition on the posttest. Consequently, the authors advocated that comparing the two convergence stories with the near miss example may promote schema induction because it highlights the structural features of the convergence stories, and thus facilitates the similarity comparison between the schema and the radiation problem to be solved in terms of the important solution features. They further suggested using confirmatory examples together with near miss examples, rather than confirmatory examples only, in teaching practice. Similar findings were obtained by studies on incorrect solutions (Curry, 2004; Van Dooren, de Bock, Hesseis, Janssens, & Verschaffel, 2004), which showed that learners having the opportunity to compare correct and incorrect solutions learned better than those who studied correct solutions only. The conclusion drawn from these studies is

that studying positive and negative examples is better than studying positive examples only. These studies, however, did not investigate the relative contribution of surface and structural similarity to the effects of positive and negative examples.

Zhang and Zhao (2008) compared effects of multiple examples with six types of surface and structural variability in helping second - grade students solve word problem. Three sets of examples varied in surface features including number and substance varied, story varied, and expression varied examples. Three sets of examples varied in structural features including rule varied, rule repeated, and rule of speed composed examples. Results showed that the three sets of superficially different examples did not better promote students' far transfer learning than one example only. Examples with different kinds of structural variability enhanced students' performance on far transfer problems, but their effects were diverse.

A recent study reported by Rittle - Johnson and Star (2009) investigated the effects of superficial similarity and structural similarity of examples on equation learning in a more systematic way. In the study, seventh - and eighth - grade students were given three types of examples to learn equation solving: comparing equivalent examples, comparing problem types, and comparing solution methods. The three types of examples differed in how the problems and solution methods varied within an example pair. As the authors stated, there are two methods of solving equation problems: the conventional distribute - first method and the composite - variable shortcut method. In the comparing equivalent examples, problems were equivalent and were solved with the same solution method; in the comparing problem types, problems were different types but were solved with the same solution method; and in the comparing solution methods, problems were the same but were solved with different methods.

Results showed that, for the most part, students who compared solution methods outperformed those who compared problem types and equivalent examples, and students who compared problem types outperformed those who compared equivalent examples, although some between - group differences were insignificant. The authors thus concluded that "moderately similar, rather than highly similar, examples should help people ignore irrelevant surface features and abstract a more general underlying solution structure" (Rittle - Johnson & Star, 2009), and recommended using contrasting examples that vary on one or a few important dimensions for learning.

Some of findings from Rittle - Johnson and Star (2009) showed that students who compared high - variability examples learned better than those who compared low - variability examples. One important implication from their study is that varying structural features of examples while keeping surface features invariant can help students ignore irrelevant surface features and induce a more general underlying structure - based schema; and this kind of comparison seems to be more effective than comparison that varies surface features while keeping the structural features invariant.

2.3 Summary

To summarize, contradictory findings have been reported regarding how similar multiple examples should be in terms of surface and structural features. In the case of examples with very dissimilar surface features, it is often more difficult for students to find the underlying common structure. Conversely, when given examples that have strong surface similarity, students may wrongly identify surface features as crucial. As Rittle - Johnson and Star (2009) concluded, it is still unclear how similar multiple examples should be and what should be compared. Sweller (2010) also pointed out that determining the appropriate level of example variability is a challenge that all teachers encounter during their routine instruction.

3 Interaction between students' prior knowledge and example variability

The effectiveness of using comparison to facilitate learning depends not only on the variability of multiple examples being compared, but also on the prior knowledge of students who compare the examples. This is captured in the question raised by Rittle - Johnson et al. (2009): "Is comparison effective early in the learning process or only after learners have sufficient prior knowledge in the domain?". Limited empirical

studies, however, have been conducted to explicitly examine this issue.

Quilici and Mayer (1996), as previously described, found in Experiment 2 that students who studied structure – emphasizing examples that had different surface stories outperformed those who studied surface – emphasizing examples that had the same surface story on the posttest of statistical problems. In Experiment 3, the authors investigated the relation between students' mathematics ability and the superficial variability of examples. Results showed that higher ability students were able to solve the statistical problems regardless of what type of examples (structure – emphasizing or surface – emphasizing) they received. In contrast, lower ability students who compared structure – emphasizing examples did better than lower ability students who compared surface – emphasizing examples.

Quilici and Mayer concluded that lower ability students encountered more difficulty in recognizing a source problem as analogous to a target problem and that this difficulty would be reduced if they received examples that emphasized structural features; comparing the structure – emphasizing examples can help lower ability students to focus on and recognize the structural features, and thus to construct more structure – based schema and demonstrate more structure – based performance. Sweller (2010) also suggested that for a specific instructional procedure to be effective, intrinsic cognitive load must be increased as long as the total cognitive load does not exceed the learners' working memory capacity. High example variability is a way to "increase intrinsic cognitive load to include interacting elements important to a task" (Sweller, 2010).

Renkl et al. (1998) examined the effect of prior knowledge on learning from comparing examples. As previously described, in Renkl et al.'s (1998) study, students were given either low – variability uniform examples with the same surface story or high – variability multiple examples with different surface stories. The authors also explored whether a learner's prior topic knowledge moderated the effects of example variability. Renkl et al. predicted that learners with higher prior topic knowledge would benefit from studying highly variable examples because they could detect the underlying common structural features of examples, whereas learners with lower prior topic knowledge would be overtaxed by high variability and thus would not benefit from highly variable examples. Results of the study, however, did not support this hypothesis. The high – variability examples condition was not better than the low – variability examples condition, as previously described, nor were the effects of example variability moderated by students' prior knowledge.

The findings of Renkl et al. (1998) did not concur with those of Quilici and Mayer (1996) with respect to what types of example variability should be provided to different students to facilitate learning, although both studies were conducted under the cognitive load theory framework. It appears complicated to determine an optimal level of example variability and intrinsic cognitive load for different learners: intrinsic cognitive load (or example variability, element interactivity) must be increased to maximize instructional effectiveness but should be decreased if it is too overwhelming to be handled by the learners' working memory (Sweller, 2010). In addition, the two studies examined two different kinds of students' characteristics, domain ability and prior knowledge, which limits the comparison of the findings.

The two studies described above focused on superficial variability rather than structural variability. A more recent study conducted by Xing and Mo (2005) examined the interaction effect between students' mathematics ability and example surface and structural variability in learning probability. They designed high variable examples as examples that had different surface and structural features, and low variable examples as examples that had similar surface and structural features. Results showed that students with high mathematics ability performed better in solving far transfer problems (accuracy and time) when given high variable examples to learn. In contrast, example variability did not differently affect the performance of students with low mathematics ability.

Rittle – Johnson et al. (2009) investigated the important role of prior knowledge in learning from comparison, as an extension of Rittle – Johnson and Star's (2009) study. Rittle – Johnson and Star (2009) found

that students comparing examples with different solution methods and the same surface features learned better than those comparing examples with the same solution methods but different surface features. Rittle – Johnson et al. (2009) further explored the effect of students' prior knowledge on the two types of example comparison. Results indicated interaction effects between students' prior knowledge and the examples being compared on their learning. In particular, students with lower prior knowledge benefited more from comparing problem types that had the same structural features but different surface features in comparison to comparing solution methods that had the same surface features but different structural features. Conversely, students with higher prior knowledge benefited more from comparing solution methods.

The authors argued that students with lower prior knowledge benefited more from comparing problem types because these examples required "fewer resources to compare problem features" and "only examining the first line of the example". Students with lower prior knowledge, however, might not have sufficient prior knowledge to benefit from comparing solutions. When given superficially similar examples that had different solution methods, students with lower prior knowledge were likely to focus on the different solution methods or structural features; however, as they did not have sufficient knowledge, those features would cause cognitive overload, "even if students noticed important features of the examples, they would need sufficient knowledge to make sense of those features".

Finally, based on findings from this study and their previous studies on learning from comparing examples, Rittle – Johnson et al. (2009) concluded that comparing methods "can be harmful for students with low prior knowledge of solution methods; neutral for students who are attempting to master one of the to – be – compared methods; and beneficial for students who accurately use one of the to – be – compared methods".

Some of the findings of Rittle – Johnson et al. (2009) are contradictory to those of Renkl et al. (1998). According to Rittle – Johnson et al.'s (2009) study, students with low knowledge benefited from comparing examples with varying surface features, which was not found by Renkl et al. (1998). More studies should be conducted to examine this inconsistent finding. In addition, Renkl et al.'s (1998) study examined superficial variability of examples, whereas Rittle – Johnson et al.'s (2009) study focused on both superficial and structural variability of examples.

In short, the studies reviewed in this section focused on different aspects of example variability and students' characteristics; their research questions, experimental conditions, and research designs were different. Thus, the relation between learners' aptitudes and example variability remains ambiguous.

4 Conclusions and further research

Although comparing multiple examples is generally good for learning, its benefit depends on the variability of multiple examples being compared and the prior knowledge of students who compare the examples. According to the contradictory findings introduced above, it seems inappropriate to divide examples into surface features and structural features, in which surface features are simply considered irrelevant and unimportant to goal attainment and structural features are considered relevant and important to goal attainment. Guo and colleagues (Guo & Pang, 2011; Guo, Pang, Yang, & Pang, 2012; Guo, Yang, & Ding, 2014) suggested using the distinction of critical/uncritical to replace the surface/structural in example study; aspects and features that are critical to students' understanding should be identified and compared in example design. When students with different prior knowledge are concerned, some surface features might become relevant and critical to learning, or some structural features might become irrelevant and uncritical to learning, i. e., students' learning should be the criterion for determining the important and relevant features.

Finally, further research should focus on more systematic arrangement about the variability of multiple examples. Most existing research has examined the effect of a limited number of examples on learning only a specific feature. Classroom teaching, however, is different. An academic topic usually involves several features or aspects that are relevant and important to student learning. For such a complex topic to be com–

pletely learned or taught in the classroom, a set of multiple examples is necessary for students to systematically compare these features in terms of similarity and difference, in one or several lessons.

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多重样例变异性与先前知识对样例学习效果的影响述评

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摘 要 样例学习研究表明多重样例的学习效果好于单个样例,因为多重样例的对比通常都有利于学习。然而并不是所有的对比都同样有效。研究者关于如何有效设计多重样例得出了不一致的结论。通过综述已有关于多重样例学习的实证研究,总结出两个尚未得到解决的问题:(1)多重样例之间的相似或相异程度应该如何;(2)学生的先前知识在多重样例学习中的作用如何。

首先,关于多重样例之间多大程度的相似或相异才能够促进学习,研究者得出了不同的结果。样例一般被认为是由表面(无关)特征和结构(相关)特征组成的。表面特征(如名称、事物、数字、表面概貌)与目标达成无关,结构特征(如数学法则、原理、解法、规则)与目标达成相关。多重样例表面和结构特征之间的相似度应该多大,已有研究尚未得出一致的结论。一方面,具有不同表面特征的多重样例能够帮助学习者注意到结构特征,提高相关认知负荷,并建构图式;表面特征很相似的多重样例有可能会模糊结构特征和表面特征之间的差异,不利于图式建构及未来的问题解决。另一方面,表面特征相似的多重样例不会增加学习者的工作记忆负担,能够帮助他们审辩、匹配结构特征;相异程度高的多重样例不利于识别样例结构上的共同点。

其次,研究者对学生先前知识在多重样例学习中的作用也没有得出一致的结论,主要有以下几种不同的观点。第一,先前知识较低的学生不能受益于多重样例对比,尤其是复杂和不熟悉的样例。第二,先前知识较高的学生能够受益于对比任何多重样例,而先前知识较低的学生只能受益于对比相异程度高的多重样例。第三,先前知识较高的学生受益于对比相异程度高的多重样例,而先前知识较低的学生则受益于对比相异程度低的样例。第四,学生的先前知识和多重样例的变异性之间不存在交互作用。

我们综述了已有研究在上述问题上的局限性,并根据自己的实证研究结果提出了改善样例设计的建议。首先,样例设计应该关注对学生学习关键的属性和特征,并使用关键/不关键的分类来代替已有认知研究采用的表面/结构的分类。其次,学生在对比多个关键属性同时变异之前需要单独审辨出每个关键属性。如果学生没有先单独审辨出每个关键属性,同时变异就可能会造成超负荷而不利于学习。最后,具有不同先前知识的学生学习时感受到不同的关键属性,多重样例应该基于学生学习时的关键属性设计特定的变易范式以帮助学生学习。

关键词 多重样例 变异性 先前知识对比