A New Forecasting Model for USD/CNY Exchange Rate

Zongwu Cai∗ Linna Chen† Ying Fang‡

*University of North Carolina at Charlotte, zcai@uncc.edu
†Xiamen University, chenlinna918@yahoo.com.cn
‡Xiamen University, yifst1@gmail.com

Copyright ©2010 The Berkeley Electronic Press. All rights reserved.
A New Forecasting Model for USD/CNY Exchange Rate*

Zongwu Cai, Linna Chen, and Ying Fang

Abstract

This paper models the return series of USD/CNY exchange rate by considering the conditional mean and conditional volatility simultaneously. An index type functional-coefficient model is adopted to model the conditional mean part and a GARCH type model with a policy dummy variable is applied to the conditional volatility model. We show that the government policy indeed has an impact on the exchange rate dynamic. To evaluate the out-of-sample forecasting ability, a prediction interval is computed by employing nonparametric conditional quantile regression. Our method outperforms other popular models in terms of various criteria.

*The authors thank the editor, Bruce Mizrach, and the referee for their valuable comments and suggestions. Cai’s research is partially supported by the National Natural Science Foundation of China #70871003 and #70971113. Fang’s research is supported by the National Natural Science Foundation of China #70971113 and by the Fundamental Research Funds for the Central Universities?#1231-ZK1001).
A New Forecasting Model for USD/CNY Exchange Rate

February 21, 2011

1 Introduction

It is commonly assumed that the exchange rate follows a martingale difference sequence (MDS) process, which implies that the future returns are unpredictable using public available information. As a result, many empirical studies in 1990s modeled exchange rates by focusing only on volatility forecasts. The most popular specification to model volatility is the generalized autoregressive conditional heteroskedasticity (GARCH) type model due to Bollerslev (1986, 1987). By incorporating the MDS hypothesis and using GARCH type models or their variants, most studies found evidence of nonlinearity in volatilities of exchange rates; see, for example, Bollerslev (1990), Brock, Hsieh and Lebaron (1991), Engle, Ito and Lin (1990), West and Cho (1995), among others.

However, by applying a generalized spectral test, Hong and Lee (2003) examined some major exchange rates in the world and found that for some exchange rates, there exist strong non-linearities in the conditional mean of exchange rates in additional to the nonlinearity in conditional volatility. This finding was advocated by Fan, Yao and Cai (2003) by using a nonparametric regression technique. Therefore, during the recent years there have been increasing interests in predicting the changes of exchange rates using nonlinear time series models. For example, Michael, Nobay and Peel (1997) employed a smooth transition autoregressive (STAR) model to analyze nonlinearities for three exchange rates. For every exchange rate examined, they rejected linearity hypothesis and found strong support for exponential STAR (ESTAR) model. Sarantis (1999) adopted a STAR model to test the nonlinearities of the real effective exchange rates for the 10 major industrial countries (the G-10). Their tests rejected the linearity hypothesis for eight out of ten industrial countries during the 1980s and 1990s. Moreover, they demonstrated some evidence that indeed, the STAR model can improve the forecasts compared to the simple random walk model, although the degree of improvement is not always unambiguous. However, the empirical evidence supporting parametric nonlinear time series models seems to be mixed. For example, Meese and Rose (1991) found that incorporation of nonlinearities into the conditional mean models does not help to improve the forecasts of the changes of exchange rates.

The advantage of modeling nonlinearities flexibly makes the nonparametric method popu-
lar in the literature. Kuan and Liu (1995) used a feed-forward and recurrent neural network model to forecast exchange rates and found a lower mean squared forecasting error (MSFE) than that in the martingale model, whereas Diebold and Nason (1990) applied nonparametric kernel regressions to analyze the nonlinearities of 10 major dollar spot rates in the post-1973 float period. Mizrach (1992) proposed a multivariate nearest-neighbor model to forecast three EMS (European Monetary System) currencies. He found that the nonparametric model was superior to the random walk only for the Lira and the improvement was not significant. Gencay (1999) investigated the predictability of spot foreign exchange rate returns from the past buy-sell signals of the simple technical trading rules by using the nearest neighbors and the feed-forward network regressions. The results indicated that the simple technical trading rules provided significant forecast improvements for the current returns over the random walk model.

Chinese foreign trade and investment, which have been started from 1978, are of capital importance in the world economy. It is well known that the US dollar versus Chinese Yuan (USD/CNY) exchange rate has been one of the most important economy indexes in the world during the last decade. Due to the specialities of Chinese economy, the whole mechanism of the CNY/USD exchange rate is different from that for the major exchange rates in the world. The study of Chinese exchange rate has been of independent research interest in the recent years due to economic and political reasons. Moreover, the USD/CNY exchange rate is changed wildly different from one period to another according to the economic reforms and policies.

In this paper, we model the dynamic of the daily USD/CNY exchange rate by considering conditional mean and conditional volatility simultaneously. Concretely, we apply the index functional-coefficient regression method proposed in Fan, Yao and Cai (2003) to model the conditional mean model of the changes of the exchange rate and a GARCH model is adopted to describe the conditional volatility, and furthermore, a nonparametric prediction interval is also provided. The functional-coefficient regression method allows more flexibility of the dynamic and can avoid the curse of dimensionality. Moreover, the nonparametric natural modeling can avoid a possible misspecification and improve the forecasting performance. One of our contributions is to find the nonlinearity both in conditional mean and conditional volatility.

This paper also contributes to the literature by successfully including policy change information into a nonparametric reduced form regression model. For example, taking advantage of the fact that the value of renminbi (RMB, the Chinese currency) is pegged to a basket of main currencies, we set a linear combination of the return series of main currencies as a smoothing variable in our functional coefficient regression model and the weight of each currency is estimated by a hybrid backfitting procedure proposed in Fan, Yao and Cai (2003). The data-driven method also confirms the choice of the smoothing variables. We also include policy dummies into the conditional mean model and the conditional volatility model as well. Various statistical tests support the inclusion of the policy dummy.
The rest of the paper is organized as follows. Section 2 introduces the background about China’s foreign exchange reforms. Section 3 presents the model and the estimation approach as well as inference methods in details. Data description and some characteristics of the daily CNY/USD exchange rate series are briefly discussed at the beginning of Section 3. Section 4 reports empirical analysis results. Section 5 compares our methods to other popular models in the literature. Section 6 concludes.

2 China’s Foreign Exchange Reforms and Its Characteristics

Since the whole mechanism of the CNY/USD exchange rate is different from that for the major exchange rates in the world such as EUR/USD, JPY/USD and KRW/USD, this section is devoted to introducing briefly the Chinese foreign exchange’s characteristics and its reforms in the recent years. Prior to the economic reform launched in 1978, Chinese trade took place within the context of the so called import substitution policy. The regime maintained an overvalued exchange rate to subsidize the import of capital goods in heavy and chemical industries. In order to maintain the overvaluation, a rigid exchange control was implemented. As described by Branstetter and Lardy (2008), key elements of the control system included a 100 percent foreign exchange surrender requirement, tight limitations on individuals to hold foreign currency, and strict controls on the outflow of foreign capital. Beginning from the early of 1980s, the Chinese government relaxed almost all of the above restrictions progressively. The USD/CNY exchange rate was 1.5 yuans (Chinese currency unit) to one dollar in 1981, and was devalued to 8.7 yuans in 1994. After a modest appreciation, the authorities fixed the exchange rate around 8.3 yuans in 1995 and kept this exchange rate until the summer of 2005. After that, the market oriented reform of foreign exchange was accelerated.

On July 21, 2005, the Chinese authority announced that the exchange rate regime would move immediately into a managed floating exchange rate regime based on market supply and demand, and furthermore the authority scrapped RMB’s peg to the US dollar, shifting to referring to a basket of main currencies to determine the value of its currency. At the same time, the USD/CNY exchange rate was raised to 8.11 from around 8.28 on that day, that is, the value of Chinese yuan was increased about 2 percent. Moreover, the daily trading price of the USD/CNY exchange rate in the inter-bank foreign exchange trading market would be allowed to float within a band of 0.3 percent around the central parity published by the central bank. The float range of RMB exchange against the dollar was raised again from 0.3 percent to 0.5 percent on May 21, 2007.

In order to further promote the flexibility of the foreign exchange, the People’s Bank of China (Chinese Central bank) decided to introduce the over-the-counter (OTC) transactions in the inter-bank spot foreign exchange market on January 4, 2006. Before the introduction
of OTC transactions, the central parity of exchange rate was determined based on the closing quotation in the inter-bank foreign exchange market. Under the system of OTC transactions, the formation mechanism of the central parity was changed. The China Foreign Exchange Trade System makes offers to all market makers before the opening of the inter-bank foreign exchange market, and the quotations of all market makers are taken except the highest and the lowest quotations. The central parity of exchange rate of RMB against US dollar for the current day is confirmed by the weighted average of all remaining quotations. The weight is determined by the China Foreign Exchange Trade System in the light of transaction volumes and the quotation conditions and other indexes.

3 The Econometric Modeling

3.1 The Data

We concern the daily USD/CNY exchange rate series from January 4, 2006 to July 18, 2008, which forms a series of 619 observations. The data are available from the Chinese State Administration of Foreign Exchange (http://www.safe.gov.cn/). Let $Z_t$ be the exchange rate on the $t$th day. Figure 1(a) shows that the price series has an obvious decreasing time trend, which reflects the fact of a gradual appreciation of RMB since 2006. Denote the return series by $Y_t = 100 \log(Z_t/Z_{t-1})$, the so-called scaled logarithmic difference. Figure 1(b) presents the time series graph of the return of exchange rate and it clearly shows a structural change on May 21, 2007. On that day, the authorities raised the float range of USD/CNY exchange rate from 0.3 percent to 0.5 percent.

![Figure 1: Time series plot of USD/CNY exchange rate](image-url)
3.2 The Model

Figure 1(b) shows that the return series is nonlinear but hard to be modeled by an existing parametric nonlinear model. However, any nonlinear model can be approximated by a nonparametric time-varying parameter or functional coefficient linear model; see Granger (2008) and Cai (2010). Following this spirit, we propose to using a flexible functional-coefficient method of Cai, Fan and Yao (2000) to model the conditional mean part and to employing the GARCH model with policy dummies to consider the conditional volatility part.

In order to forecast the return series of exchange rate, we are interested in estimating $E(Y_t | I_{t-1})$, where $I_{t-1}$ is the information set available at time $t - 1$, that is, $I_{t-1} = \{Y_{t-1}, \ldots, Y_1, X_{t-1}, \ldots, X_1\}$, where $X_{t-1}$ denotes other available explanatory variables. The models can be summarized as follows:

$$Y_t = \mu_t + \eta_t, \quad \text{where} \quad \eta_t = \sigma_t \varepsilon_t. \quad (3.1)$$

For the conditional mean part $\mu_t$, it is ideal to model $\mu_t$ by a nonparametric form as $\mu_t = \mu(I_{t-1})$ but it suffers from the so-called “curse of dimensionality” due to the high dimensional modeling. To overcome this problem, we employ a functional-coefficient regression model, in which the coefficients depend on some smoothing variables $U_t$. The functional-coefficient regression model imposes very little model assumptions but allows appreciable flexibility on the model structure, and furthermore, the additive structure can effectively avoid the “curse of dimensionality” in nonparametric regressions. A functional-coefficient regression model can be defined by

$$\eta_t = E(Y_t | U_t, X_t) = \sum_{j=1}^{p} a_j(U_t)X_{tj}, \quad (3.2)$$

where, $Y_t \in \mathbb{R}^1$ is a dependent variable, $X_t \in \mathbb{R}^p$ are explanatory variables and $U_t \in \mathbb{R}^k$ are smoothing variables. We assume that $\{Y_t, X_t, U_t\}_{t=1}^{\infty}$ are strictly stationary and $\{a_j(\cdot)\}_{j=1}^{p}$ are measurable functions mapping from $\mathbb{R}^k$ to $\mathbb{R}^1$; see Cai, Fan and Yao (2000) for details.

For the conditional volatility part $\sigma_t$, a GARCH type model is used with policy dummy variable. To address the structural change occurred on May 21, 2007, a policy dummy variable $C_t$ is considered, which takes a value of zero for observations before that day and one for the remaining observations. That is,

$$C_t = \begin{cases} 
0, & \text{when} \quad t < 20070521; \\
1, & \text{when} \quad t \geq 20070521. 
\end{cases}$$

We also add the same policy dummy variable in the conditional mean part. By combining
(3.1) and (3.2), the forecasting model takes the following form:

\[ Y_t = a_C(U_t)C_t + \sum_{j=1}^{p} a_j(U_t)Y_{t-j} + \eta_t = \sum_{j=1}^{p+1} a_j(U_t)X_{tj} + \sigma_t \varepsilon_t, \quad (3.3) \]

\[ \sigma_t^2 = \gamma_0 + \gamma_1 \eta_{t-1}^2 + \delta_1 \sigma_{t-1}^2 + \alpha_t C_t, \quad (3.4) \]

with \( \gamma_0 > 0, \gamma_1 \geq 0, \delta_1 \geq 0, \) and \( \gamma_1 + \delta_1 < 1, \) where \( X_{tj} = Y_{t-j}, \) \( X_{t,p+1} = C_t, \) \( U_t \) is a univariate smoothing variable determined later and \( \{a_j(\cdot)\}, 1 \leq j \leq p + 1, \) are continuous functions.

### 3.3 The Nonparametric Estimation

There are several nonparametric estimation techniques available for estimating the functional coefficient \( \{a_j(\cdot)\}. \) Here we employ the local linear regression method due to its attractive properties such as the boundary correction and minimax efficiency; see Fan (1993) and Fan and Gijbels (1996). We assume throughout that \( a_j(\cdot) \) has a continuous second derivative. Then, for any given grid point \( u_0 \in \mathbb{R}^k, \) when \( U_t \) is in a neighborhood of \( u_0, \) \( a_j(U_t) \) is approximated locally at \( u_0 \) by the Taylor expansion. That is, \( a_j(U_t) \approx a_j(u_0) + a_j(u_0)^\top (U_t - u_0), \) where \( \hat{a}_j(u_0) = \partial a_j(u_0)/\partial u_0. \) The local linear estimate is defined as \( \hat{a}_j(u_0) = \hat{a}_j, \) \( \hat{b}_j(u_0) = \hat{b}_j, \) where \( (\hat{a}_j, \hat{b}_j) \) minimizes the sum of locally weighted squares:

\[
\sum_{t=1}^{n} \left[ Y_t - \sum_{j=1}^{p+1} (a_j + b_j^\top (U_t - u_0)X_{tj}) \right]^2 K_h(U_t - u_0),
\]

where \( K_h(\cdot) = h^{-k}K(\cdot/h), \) \( K(\cdot) \) is a kernel function on \( \mathbb{R}^k, h > 0 \) is a bandwidth, and \( h \to 0 \) as \( n \to \infty. \) By moving \( u_0 \) along the whole domain of \( U_t, \) the entire estimated surface of \( a_j(u_0) \) is obtained.

The data-driven fashion and optimal bandwidth is chosen by the nonparametric Akaike information criterion (AIC) developed by Hurvich, Simonoff and Tsai (1998) and Cai and Tiwari (2000), which is specifically designed as an approximately unbiased estimator of the expected Kullback-Leiber information criterion in nonparametric regression settings. Also, the nonparametric AIC can be used for choosing the “optimal” order \( p \) in model (3.3); see Cai and Tiwari (2000) for details.

Besides the above local linear estimation method, Huang and Shen (2004) proposed a global smoothing method based on a polynomial spline to estimate the functional-coefficient model. One appealing feature of their method is that the functional coefficients can depend on different smoothing variables. However, the local linear estimation has the advantage of utilizing local information sufficiently and can obtain the minimax efficiency (Fan, 1993).
3.4 The Selection of Smoothing Variable

It is important to choose an appropriate smoothing variable $U_t$ when applying the functional-coefficient regression model. Knowledge on physical background of the data or economic theory and events may be very helpful. Some data-driven methods to choose the smoothing variables are also available, such as the Akaike information criterion, cross-validation, and other criteria; see Cai, Fan and Yao (2000) and Fan, Yao and Cai (2003). We benefit from the fact that the value of RMB pegs to a basket of main currencies. The Governor of the People’s Bank of China, Mr. Xiaochuan Zhou, stated on August 9, 2005 that the major currencies in the basket include US dollar (USD), Euro (EUR), Japanese Yen (JPY), and Korean Won (KRW). Therefore, we choose the smoothing variable as a linear combination of the return series of three major exchange rates: EUR/USD, KRW/USD and JPY/USD as

$$U_t = \beta_{EUR} \cdot r_{EUR,t} + \beta_{KRW} \cdot r_{KRW,t} + \beta_{JPY} \cdot r_{JPY,t}, \quad (3.5)$$

where $\beta = (\beta_{EUR}, \beta_{KRW}, \beta_{JPY})$ are weights satisfying the identification condition $\beta_{EUR} + \beta_{KRW} + \beta_{JPY} = 1$, and $r_{EUR,t}$, $r_{KRW,t}$ and $r_{JPY,t}$ are the return series of EUR/USD, KRW/USD and JPY/USD, respectively. The weights can be automatically determined by the data because they are unknown. Notice that model (3.3) becomes to the index functional coefficient model proposed in Fan, Yao and Cai (2003).

The weights $\beta$ are estimated by a hybrid backfitting algorithm method proposed by Fan, Yao and Cai (2003). Basically, the estimation method is an alternating iteration between estimating the linear index through a one-step scheme proposed by Bickel (1975) and estimating the functional coefficients through an one-dimensional local linear smoothing method. To minimize the $E(Y - \hat{Y})^2$, one should search for $\beta$ to minimize

$$R(\beta) = \frac{1}{n} \sum_{t=1}^{n} \left( Y_t - \sum_{j=1}^{p+1} a_j(U_t)Y_{t-j} \right)^2 w(U_t),$$

where $w(\cdot)$ is a known weighting function. Suppose that $\hat{\beta}$ is the minimizer of the above equation, then $\hat{R}(\hat{\beta}) = 0$, where $\hat{R}(\cdot)$ denotes the derivative of $R(\cdot)$. For any $\beta^{(0)}$ close to $\hat{\beta}$, we have the Taylor approximation $0 = \hat{R}(\hat{\beta}) \approx \hat{R}(\beta^{(0)}) + \hat{R}(\beta^{(0)})(\hat{\beta} - \beta^{(0)})$, where $\hat{R}(\cdot)$ is the Hessian matrix of $R(\cdot)$. This leads to the one-step iterative estimate similar to the Newton-Raphson procedure:

$$\beta^{(1)} = \beta^{(0)} - \hat{R}(\beta^{(0)})^{-1} \hat{R}(\beta^{(0)}),$$

where $\beta^{(0)}$ is the initial value. At each iteration, we re-scale $\beta^{(1)}$ such that it has unit sum. We refer the reader to the paper by Fan, Yao and Cai (2003) for details.

Alternatively, one might consider other smoothing variables used in the literature, such as the moving average technique trading rule (MATTR)

$$U_{t,MATTR} = \frac{Y_{t-1}}{\sum_{j=1}^{L} Y_{t-j}/L} - 1$$
for certain $L$ (say, $L = 21$), as in Brock, Lakonishock and Lebaron (1992) and Hong and Lee (2003). Indeed, $U_{t,MATTR}$ has a nice economic interpretation; see the aforementioned papers for details. Based on our empirical study, we find that the linear combination of several main currencies versus US dollar outperforms MATTR in terms of the nonparametric AIC criterion.

3.5 A Goodness-of-Fit Test

It is important to consider the goodness-of-fit of the nonparametric model proposed above. For example, it is interesting to test whether the policy dummy variable is significant.

Firstly, we test for the linear regression model against the nonparametric functional-coefficient regression model by employing the method proposed by Cai, Fan and Yao (2000). The null hypothesis is defined as:

$$H_0 : a_j(U_t) = \alpha_j \quad \text{for} \quad 1 \leq j \leq p + 1, \quad (3.6)$$

so that model (3.3) becomes a $p$th order autoregressive model with an exogenous variable $C_t$, where $\{a_j\}$ are constant parameters in the AR(p). In other words, $U_t$ does not have an impact on the USD/CNY exchange rate. In particular, it is interesting to see if the policy dummy variable is significant. That is equivalent to testing the null hypothesis defined as

$$H_0 : a_C(U_t) = 0, \quad (3.7)$$

which is to test a full nonparametric functional-coefficient regression model against a reduced nonparametric functional-coefficient regression model. To consider the test in (3.6) and (3.7), one can apply a generalized likelihood ratio (or generalized F-type) test proposed by Cai, Fan and Yao (2000) and studied by Fan, Zhang and Zhang (2001), which can be constructed as

$$F = \frac{RSS_0 - RSS_1}{RSS_1} = \frac{RSS_0}{RSS_1} - 1,$$

where $RSS_0$ and $RSS_1$ are the residual sum of squares under the null and alternative hypotheses respectively. The null hypothesis is rejected for large value of the test statistic. To calculate the $p$-value, a nonparametric wild bootstrap approach can be employed; see Cai, Fan and Yao (2000) for details.

3.6 Prediction Intervals

It is easy to see from (3.3) and (3.4) that the $\tau$th conditional quantile of $Y_t$ given $U_t$ and $X_t$ is

$$q_{\tau}(U_t, X_t) = \sum_{j=1}^{p+1} a_j(U_t)X_{tj} + \sigma_t F^{-1}_\tau(\tau), \quad (3.8)$$
where $F_\varepsilon(\cdot)$ is the distribution of $\varepsilon$. Therefore, a naive $(1 - \alpha)100\%$ prediction interval can be constructed as

$$
\left( \sum_{j=1}^{p+1} \hat{a}_j(U_t)X_{tj} - \hat{\sigma}_1 F_\varepsilon^{-1}(\alpha/2), \sum_{j=1}^{p+1} \hat{a}_j(U_t)X_{tj} + \hat{\sigma}_1 F_\varepsilon^{-1}(1 - \alpha/2) \right).
$$

To make prediction interval in a nonparametric nature, the above quantile regression function in (3.8) is generalized to be more a general form as

$$
q_r(U_t, X_t) = a_{C_r}(U_t)C_t + \sum_{j=1}^{p} a_{j,r}(U_t)X_{tj},
$$

which was proposed by Cai and Xu (2008), where \{\{a_{j,r}(\cdot)\}\} might depend on $r$. To estimate \{\{a_{j,r}(\cdot)\}\} nonparametrically, one can use a nonparametric quantile regression estimation procedure as in Cai and Xu (2008). Use the Taylor expansion,

$$
q_r(U_t, X_t) \approx \hat{\beta}_{0,r}^T X_t + \hat{\beta}_{1,r}^T X_t(U_t - u_0),
$$

and then find $(\hat{\beta}_{0,r}, \hat{\beta}_{1,r})$ to minimize the following

$$
\sum_{t=1}^{n} \rho_r \left( Y_t - \hat{\beta}_{0,r}^T X_t - \hat{\beta}_{1,r}^T X_t(U_t - u_0) \right) K_h(U_t - u_0),
$$

where $\rho_r(z) = z(\tau - I_{\{z < 0\}})$ is the loss function and $I_A$ denotes the indicator function of any set $A$. Then, the nonparametric estimation of $a_{j,r}(u_0)$ is the $j$th element of $\hat{\beta}_{0,r}$, so that the nonparametric estimation of $q_r(u_0, X_t)$ is $\hat{\beta}_{0,r}^T X_t$. By changing the value of $\tau$ from 0 to 1, a set of quantile regressions are obtained. Particularly, when letting $\tau_1 = 0.025$ and $\tau_2 = 0.975$, then a 95% prediction interval is obtained and it is $(\hat{q}_{\tau_1}(U_t, X_t), \hat{q}_{\tau_2}(U_t, X_t))$.

4 Empirical Results

4.1 Policy Change and Structural Breaks

Although the functional coefficient model defined in (3.2) is sufficiently to model continuously time varying features of the exchange rate by assuming that the time varying coefficients depend on a smooth function of other random variables, it would be still helpful to include policy changes or discrete structural breaks into the functional coefficient regressions. We notice that there exists an important policy change during the sample period. On May 21, 2007, the float range of RMB exchange rate against the US dollar was raised from 0.3 percent to 0.5 percent. Figure 1(b) demonstrates that larger daily volatility of returns of exchange rate can be observed after that date than that before May 21, 2007. We include a policy dummy $C_t$, which takes a value of zero before that date and one on or after that date, into the functional coefficient regression.

The sup Wald test (Andrews, 1993) detects a break at the date of October 16, 2007, which is about 5 months later than the policy change date. A break dummy is also defined by
taking a value of zero before October 16, 2007 and one on or after that date. Now we consider two competing models: one is the model in (3.3) with a policy dummy and the other is the model including both policy dummy and break dummy. The encompassing (ENC-NEW) test (Clark and McCracken, 2001) is employed to test whether the model including the break dummy can improve the post sample forecast accuracy. The null hypothesis is that the nested model is better than the model in the alternative which includes both dummies. The p-value of the test is about 0.86 then we can not reject the null. We also compute the mean square forecasting errors (MSFEs) and mean average forecasting errors (MAFEs) for four alternative models: (1) a functional coefficient model with a policy dummy; (2) a functional coefficient model without dummies; (3) a functional coefficient model with a break dummy; and (4) a functional coefficient model with both dummies. The results conclude that the functional coefficient model with a policy dummy dominates all others in terms of post sample forecasting errors.

4.2 Estimation Results

For the conditional mean part, we use an index functional-coefficient regression model with the coefficients depending on the smoothing variable $U_t$ given in (3.5). By using the nonparametric AIC method and the hybrid backfitting algorithm, the selected optimal bandwidth is $h = 1.2$, the selected optimal lag term is $p = 2$ and the selected optimal weights are $\hat{\beta} = (0.2967, 0.3843, 0.3190)$. The smoothing variable is constructed by a linear combination of the return series of EUR/USD, KRW/USD and JPY/USD with the estimated weights $\hat{\beta}$ and its time series plot is presented in Figure 2. The estimated functional coefficient curves are displayed in Figure 3. Finally, both goodness-of-fit tests for testing constant coefficients

![Smoothing Variable](image)

Figure 2: Time series plot of smoothing variable series.

---

1 The MSFEs for all models are 0.0141, 0.0150, 0.0148 and 0.0151, respectively. The MAFEs are 0.1016, 0.1023, 0.1046 and 0.1055, respectively.
and testing significance of policy variable are considered. The p-values are 0.048 and 0.001, respectively, so that the coefficients are nonconstant and the policy variable is significant. Therefore, there exists a nonlinearity in the dynamic of USD/CNY exchange rate series and the government policy has an impact on the currency exchange.

For the conditional volatility part, we employ a GARCH model with a policy dummy variable $C_t$. The estimation results are reported as follows:

$$Y_t = \mu_t + \eta_t,$$

where $\eta_t = \sigma_t \varepsilon_t$,

$$\sigma_t^2 = 0.0010 + 0.1880 \eta_{t-1}^2 + 0.73589 \sigma_{t-1}^2 + 0.0020 C_t$$

$p$-value (0.0000) (0.0000) (0.0000) (0.0000).

The coefficient of the policy dummy variable $C_t$ is 0.0020 with associated $p$-value 0.0000, which is highly significant. This result coincides with our expectation that the increase of floating range has a significant impact on the mechanism of generating volatilities.

### 4.3 Prediction Intervals

Finally, to evaluate the performance of our model, we compute a 95\% prediction interval using nonparametric quantile estimation. The results are presented in Figure 4, which demonstrates the out-of-sample forecasting results for the last 50 days. From Figure 4, there are only two sample points (out of 50 sample points) outside the 95\% prediction interval. This implies that the out-of-sample forecasting performance of our model performs reasonably well.

### 5 Comparisons with Other Models

There are several models commonly used in the forecast of exchange rates in the literature. For example, an incomplete list of these models includes the random walk model (RW), the autoregressive model, the threshold autoregressive model (TAR), the smooth transition
autoregressive model (STAR), the artificial neural network model (ANN) and the functional coefficient regression model using a polynomial spline proposed by Huang and Shen (2004, denoted by HS). The forecasting performances of these models seem to be mixed. Meese and Rogoff (1983) compared the out-of-sample forecasting accuracy of various structural and time series exchange rate models. They found that a random walk model performs as well as any estimated model due to sampling error, while Michael, Nobay and Peel (1997) and Sarantis (1999) used parametric STAR model to analyze the nonlinearities for exchange rates and rejected the linearity hypothesis for the real effective exchange rates.

Furthermore, we compute the out-of-sample one-step-ahead forecasts to compare our model with other models in terms of MSFE and MAFE, defined as follows:

\[
MSFE = m^{-1} \sum_{i=1}^{m} (Y_{T+i} - \hat{Y}_{T+i})^2, \quad \text{and} \quad MAFE = m^{-1} \sum_{i=1}^{m} |Y_{T+i} - \hat{Y}_{T+i}|, 
\]

where \( m \) is the forecasting period and \( m = 50 \). It has been well documented that the random walk model often outperforms complicated structural time series models in forecasting the conditional mean of exchange rate changes. The MSFE and MAFE for our model and other models are listed in Table 1.

Table 1 demonstrates that both MSFE and MAFE for our model is smaller than those of alternative models. Since the difference of MSFE and MAFE between our model and the random walk model is small, to check whether the difference is significant, the superior predictive ability (SPA) tests (White, 2000; Hansen, 2005) are employed. The null hypothesis is that the proposed model is not inferior to all alternative models which include the random walk model, the threshold autoregressive model, the smooth transition autoregressive model, the artificial neural network model and the functional coefficient model using
Table 1: A comparison of MSFE and MAFE values for all models

<table>
<thead>
<tr>
<th></th>
<th>FC</th>
<th>RW</th>
<th>TAR</th>
<th>STAR</th>
<th>ANN</th>
<th>HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSFE</td>
<td>0.0141</td>
<td>0.0153</td>
<td>0.0176</td>
<td>0.0221</td>
<td>0.0155</td>
<td>0.0295</td>
</tr>
<tr>
<td>MAFE</td>
<td>0.1016</td>
<td>0.1033</td>
<td>0.1091</td>
<td>0.1167</td>
<td>0.1031</td>
<td>0.1273</td>
</tr>
</tbody>
</table>

NOTE: FC, RW, TAR, STAR, ANN and HS denote the functional coefficient model proposed in our paper, the random walk model, the threshold autoregressive model, the smooth transition autoregressive model, the artificial neural model and the functional coefficient model using a polynomial spline method, respectively. The MSFE denotes the mean square forecasting error and the MAFE denotes the mean absolute forecasting error.

We adopt the distance of squared forecasting errors (SFE) and the distance of absolute forecasting errors (AFE) respectively as the loss function to evaluate model performance. SPA_d is the reality check test proposed by White (2000), and SPA_l and SPA_u denote the superior predictive ability tests proposed by Hansen (2005). The SPA_l test simply deletes all poor alternatives in the test statistic, while the SPA_u test keeps alternative models with moderately poor performance. The latter test usually has better finite sample performance than the former because the SPA_u test accounts for the fact that poor alternatives may have an impact on the critical value. The p-values of all tests are presented in Table 2. All tests can not reject the null hypothesis that our model is not inferior to any of the alternatives.

Table 2: P-values of superior predictive ability tests

<table>
<thead>
<tr>
<th></th>
<th>SPA_d</th>
<th>SPA_u</th>
<th>SPA_u</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFE</td>
<td>0.1700</td>
<td>0.7290</td>
<td>0.9010</td>
</tr>
<tr>
<td>AFE</td>
<td>0.4808</td>
<td>0.7888</td>
<td>0.9084</td>
</tr>
</tbody>
</table>

NOTE: SPA_d denotes the reality check test (White, 2000) and SPA_c and SPA_u denote the superior predictive ability tests (Hansen, 2005). The row of SFE lists the p-values that are computed based on the square forecasting errors. The row of AFE lists the p-values that are computed based on the absolute forecasting errors. The null hypothesis is that our model is not inferior to all alternative models.

Finally, we apply the ENC-NEW tests proposed in Clark and McCracken (2001, 2005) to compare the equal forecast accuracy among all alternative models. For each ENC-NEW test, we put one of competing models (RW, TAR, STAR, ANN and HS) in the null and the functional coefficient model proposed by our paper is in the alternative. Table 3 shows that the p-values of all tests are very close to zero and demonstrates a significant evidence to

---

2For the HS model, a cubic spline is adopted and the number of knots is selected by minimizing the AIC. The selected number is 5 and and all knots are placed with equal distance.
reject the null models.

<table>
<thead>
<tr>
<th>ENC-New</th>
<th>RW</th>
<th>TAR</th>
<th>STAR</th>
<th>ANN</th>
<th>HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample values of statistics</td>
<td>3.9377</td>
<td>13.827</td>
<td>31.017</td>
<td>4.449</td>
<td>50.884</td>
</tr>
<tr>
<td>p-value</td>
<td>4.11E-05</td>
<td>0.0000</td>
<td>0.0000</td>
<td>4.31E-06</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

NOTE: FC, RW, TAR, STAR, ANN and HS denote the functional coefficient model proposed in our paper, the random walk model, the threshold autoregressive model, the smooth transition autoregressive model, the artificial neural model and the functional coefficient model using a polynomial spline method, respectively. The 2nd to the 6th columns denote different models in the null hypothesis and our model is in the alternative hypothesis.

6 Conclusion

We model the conditional mean and conditional volatility of the return series of USD/CNY exchange rate simultaneously. A functional-coefficient index model is adopted to model the conditional mean part and a GARCH (1,1) model with a policy dummy is applied to the conditional volatility model. A prediction interval is computed by employing nonparametric conditional quantile regressions. Our method outperforms other popular models in terms of mean squared forecasting errors and absolute forecasting errors.

Since the Chinese government made several reforms on the daily USD/CNY exchange rate, so that the policy changes should have an impact on the dynamic of the exchange rate. We include policy dummy variables into the conditional mean model and the conditional volatility as well. The daily float limit of RMB exchange against USD was relaxed from 0.3 percent to 0.5 percent on May 21, 2007. A dummy corresponding to the policy change was included and the tests show that the dummy is highly significantly both in the conditional mean and the conditional volatility models, which provides evidence that gradual liberalization can improve the flexibility of Chinese foreign exchange rate.

Moreover, the goodness-of-fit test demonstrates that the functional coefficients is significantly nonconstant, varying with a linear combination of other currencies which includes the Euro, Japanese yen and Korean won. Using the changes of the values of other currencies is helpful to predict the dynamics of the USD/CNY exchange rate. Our empirical results partially support the recent findings of Frankel and Wei (2007). They found a modest but steady increase in weights of non-dollar currencies in the basket to determine the value of RMB, which would increase the flexibility of China’s exchange rate.
References


